Coupled Heat and Helium Transport in Groundwater

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Chapter 1

Introduction

Heat and helium (He) are often used as natural tracers in groundwater flow systems [e.g., Sorey, 1971; Taniguchi, 1993; Torgersen et al., 1994; Hilton, 1996; Castro 1998a; Pribnow & Schellschmidt, 2000]. In geothermal reservoirs and volcanic regions, geoscientists typically utilize heat and helium from the mantle and intruding magma in order to evaluate geothermal resources or constrain the size of volcanic plumbing systems [e.g., Lupton, 1983; Ingebritsen et al., 1992; Torgersen 1993; Manga & Kirchner, 2004].

The dynamics of heat and groundwater transport have been studied extensively, for example by Stallman [1963], Bredehoeft & Papadopulos [1965], Domenico & Palciauskas [1973], Mansure & Reiter [1979], Smith & Chapman [1983], Woodbury & Smith [1985], Forster & Smith [1988b], Clauser & Villinger [1990], López & Smith [1995], Bächler et al. [2003], and Saar & Manga [2004]. The use of helium as a tracer of fluid flow is also well documented in studies by Mazor & Wasserburg [1965], Truesdell et al. [1977], Mazor [1978], Kennedy et al. [1985], Kennedy et al. [1987], Hearn et al. [1990], Marty et al. [1993], O’Nions & Ballentine [1993], Hilton [1996], Cserepes & Lenkey [1999], Bethke et al.
[1999], James et al. [2000], Ballentine & Lollar [2002], and Kulongoski et al. [2005].

A large number of investigations concerning both heat and helium transport exists; however, they are not exhaustive. Commonly these are case studies, for example, those by Stute et al. [1992], Torgersen et al. [1994], Torgersen et al. [1995], Castro et al. [1998a], Clauser et al. [2002], Castro et al. [2005], and Saar et al. [2005]. Generalized, one-dimensional quantitative analyses of heat and helium transport include Bickle & McKenzie [1987], Torgersen [1993], and Bach et al. [1999]. However, there are currently no general studies for geothermally active regions that utilize coupled transport of heat and helium to investigate the effects of aquifer and tracer characteristics on near-surface heat and helium signals.

When relative contributions of heat and helium sources are constrained, they can provide important information that can be difficult to obtain by other means about the source and transport paths of groundwater. For example, groundwater in deep, large-scale flow systems often has residence times too large for the use of $^{14}$C dating [Stute et al., 2002] and anthropogenic tracer analysis [e.g., Doctor et al., 2006; Ford & Williams, 2007].

Because magmatic intrusions and the Earth’s mantle provide both heat and helium to the crust, a first-order assumption is that the near-surface signals of these natural tracers should be coupled, or that the signals should coincide in both space and time. However, temperature and magmatic helium signals are sometimes spatially and/or temporally decoupled near the surface, for example in the Rhine Graben in southwestern Germany [Griesshaber et al., 1992; Clauser et al., 2002], the Oregon Cascades in the northwestern United States [Evans et al., 2004; Saar et al., 2005], and in the Canadian Cordillera in southwestern Canada [Clark & Phillips, 2000].
The primary impetus behind this thesis is the question of what causes decoupling of magma-sourced heat and helium signals in the upper portion of the Earth’s crust. In the process of answering this question, new ones came to light. The product of the research is intended to be a resource for heat and helium research. In this contribution, I present background information about heat, helium, and groundwater transport (Chapter 2). I continue by discussing special concerns involved with choosing boundary conditions for heat and helium (Chapter 4). Chapter 4 also introduces the Hybrid Boundary Condition, a method that allows values along the surface of a model to vary according to the rate and pattern of fluid flow in the system. Chapter 3 includes examples of coupled heat, helium, and groundwater transport models and a discussion of how these relate to our current understanding of the process. The effect of considering temperature-dependent helium diffusion on concentration-depth profiles is also introduced in this chapter. I conclude by enumerating potential research projects in Chapter 6 that will increase our ability to use heat and helium to characterize geothermal reservoirs.
Chapter 2

Background

This chapter provides a brief introduction to concepts commonly employed for modeling and analysis of heat and helium in groundwater flow systems.

2.1 Transport Equations

The transfer of an incompatible solute in a single-phase fluid through a porous medium by diffusion and advection without source or sink terms is described by the advection-diffusion equation,

$$\nabla \cdot (\rho \phi \tau \mathbf{D} \cdot \nabla C) - \nabla \cdot (\rho \phi \mathbf{u} C) = \frac{d(\rho \phi C)}{dt},$$

(2.1)

where $\rho$ is the fluid density, $\phi$ is the connected pore fraction, $\tau$ is the pore-space tortuosity, $\mathbf{D}$ is the second-rank diffusion coefficient tensor of the dissolved solute in the fluid of interest, $\mathbf{C}$ is the solute concentration, expressed as a dimensionless mass fraction, $\mathbf{u}$ is the seepage velocity vector, and $t$ denotes time. One-dimensional (1D) vertical flow in a homogeneous
medium reduces Equation (2.1) to

$$\frac{\partial}{\partial z} \left( \rho D_{e,z} \frac{\partial C}{\partial z} \right) - \phi u_z \frac{\partial (\rho C)}{\partial z} = \rho \frac{d(\rho C)}{dt},$$

(2.2)

where $z$ denotes the vertical direction and $D_{e,z} = \phi \tau D_z$ is the effective diffusion coefficient ($D_e$), a product of the molecular diffusion coefficient in water and the tortuosity and porosity of the medium.

Diffusive and advective mass fluxes per unit area [kg/(m$^2$ s)] along the $z$-axis are calculated by

$$q_{D,z} = -\rho D_{e,z} \frac{\partial C}{\partial z}$$

(2.3)

and

$$q_{A,z} = -\rho k_z \left( \frac{\partial P}{\partial z} - \rho g_z \right) C = \rho \phi u_z C,$$

(2.4)

respectively, where $k_z$ is the vertical permeability, $\mu$ is dynamic fluid viscosity, $P$ is pore-fluid pressure, $\phi u_z$ is the vertical Darcy velocity or specific discharge, and $g_z$ is gravitational acceleration.

Conduction and convection of heat in a porous medium without source or sink terms is described by the equation

$$\nabla \cdot (\lambda \nabla T) - \nabla \cdot (\rho c \phi u T) = \frac{d(\rho c T)}{dt},$$

(2.5)

where $\lambda$ is the thermal conductivity of the solid-fluid matrix, $T$ is temperature, and $c$ is the specific heat of the fluid. Conductive and convective heat flow per unit area (W/m$^2$) along the $z$-axis are described by

$$d_{D,z} = -\lambda \frac{\partial T}{\partial z}$$

(2.6)

and

$$d_{A,z} = \rho c \phi u_z T,$$

(2.7)
respectively.

2.2 Characterizing Transport

Significant advective transport of a tracer, whether heat or a solute, occurs when the component of fluid flow parallel to the concentration gradient is sufficiently high to influence the distribution of the tracer. In general, the presence of vertical fluid flow creates nonlinear concentration-depth profiles. This nonlinearity can be represented by the dimensionless Peclet number, \( Pe \). \( Pe \) is essentially the ratio of bulk fluid flow to diffusive mass flux of a tracer and, for 1D, vertical solute transport, is expressed as

\[
Pe_{1D} = \frac{\phi u_z L}{D_{e,z}},
\]

(2.8)

where \( L \) is a characteristic length scale, usually the height of the fluid-transmitting layer of interest.

While the formulation provided in Equation 2.8 is useful for understanding 1D transport regimes, multi-dimensional systems require an alternate formulation that considers the geometry of the domain and the effects of forced (e.g., topography-driven) advection. Domenico & Palciauskas [1973] derived a modified version of the Peclet number,

\[
Pe_{2D} = \left(\frac{KB}{D_e}\right) \left(\frac{z_0}{x_0}\right),
\]

(2.9)

that includes these considerations for two-dimensional (2D) systems. Here, \( z_0 \) is the minimum height of the saturated zone, \( x_0 \) is the width of the system, \( B \) is the mean elevation of the water table above \( z_0 \), and \( K = \rho g_z k / \mu \) is the hydraulic conductivity of an isotropic, homogeneous porous medium with permeability, \( k \). \( Pe_{2D} \) is analogous to \( Pe_{1D} \), in that,
instead of using the velocity of the fluid directly, the advective portion of the ratio is represented by $KB$. Further, rather than defining a characteristic length for uni-directional flow, the aspect ratio ($z_0/x_0$) of the system is considered.

$Pe_{1D}$ is advantageous for characterizing one-dimensional flow. Bredehoeft & Papadopulos [1965] employ this method for type-curve matching of temperature-depth profiles in a semi-confining layer. A similar approach can be applied to concentration-depth profiles and solute transport [e.g., Bickle & McKenzie, 1987]. Alternatively, $Pe_{1D}$ can be used to estimate the relative importance of advective to diffusive solute flux as a function of the Darcy velocity, system dimension, and diffusion coefficient of the solute.

The use of $Pe_{2D}$ is useful for determining the effect of permeability and system geometry on the relative importance of advective and diffusive transport of a solute at a large regional scale. For example, concentration patterns in wider and shallower systems are more likely to be controlled by diffusive processes [Domenico & Palciauskas, 1973], as indicated by smaller $z_0/x_0$ ratios in Equation 2.9.

### 2.3 Helium Diffusion

Unlike heat diffusivity, the diffusion coefficient of helium depends significantly on temperature (Figure 2.1a), varying by nearly one order of magnitude over a temperature range of $0 \leq T \leq 200^\circ$C [Ohsumi & Horibe, 1984; Jähne et al., 1987].

Jähne et al. [1987] performed experiments to determine the diffusion coefficients of helium, neon, krypton, xenon, H$_2$, CH$_4$, and CO$_2$. The curves plotted in Figure 2.1b are based on that same study, with values for argon extrapolated by Ballentine et al. [2002]. In this contribution, I employ the equation for temperature-dependent helium diffusion
Figure 2.1: (a) The diffusion coefficient of helium as a function of temperature. Note the line at 35°C; this is the highest temperature at which the rate of helium diffusion was measured. (b) Diffusion of helium relative to other noble gases, all as a function of temperature. [Jähne et al., 1987]

coefficients provided by Jähne et al. [1987],

$$D(^4\text{He}) = 8.180 \times 10^{-7}[\text{m}^2/\text{s}] \times \exp\left(\frac{-1407\text{K}}{T}\right),$$

(2.10)

where $T$ is temperature in Kelvin (K). Because Jähne et al. [1987] performed these experiments to a maximum temperature of 35°C, care should be taken in employing these equations for temperatures that extend beyond the experimental range.

Jähne et al. [1987] also mention that because $^3\text{He}$ is lighter than $^4\text{He}$, there ought to be some variation in their diffusion coefficients. Theoretically, $^3\text{He}$ should have a diffusion coefficient that is 13% higher than $^4\text{He}$, an amount proportionate to the mass difference of the two isotopes. If this is the case, it would certainly have an impact on the value of isotope ratios through the system, which subsequently would influence analysis of helium data in that system. However, Jähne et al. [1987] provide only one data point to support the assertion that such a large mass-dependent variation in helium diffusion coefficients exists. Additionally, as temperature increases, elemental mass becomes less important in
the determination of diffusion rates (Figure 2.1b). While the mass-dependence of helium diffusion may be important in some circumstances, particularly at lower temperatures, there is currently insufficient experimental data to support this. Therefore, the mass-dependence of helium diffusion warrants mentioning, but at present there is not enough data to begin an investigation on the effects of mass-dependent diffusion on helium signals in groundwater.

2.4 Helium Isotope Ratios

Because helium is an incompatible solute, it is an excellent tracer for determining groundwater flow. It does not adsorb onto mineral surfaces nor does it interact with other chemical species in solution.

The isotope ratio of helium concentration is commonly expressed as

\[ R = \frac{^{3}\text{He}}{^{4}\text{He}}, \]

where concentration is given in cm\(^3\)/g(H\(_2\)O) under 25\(^\circ\)C and 1 atm, or standard temperature and pressure (STP). The atmospheric value of \( R_a \approx 1.4 \times 10^{-6} \) [Clarke et al., 1976; O’Nions & Oxburgh, 1983; Mamyrin & Tolstikhin, 1984] is typically used to normalize ratios, such that

\[ \frac{R}{R_a} = \frac{^{3}\text{He}/^{4}\text{He}}{^{3}\text{He}_a/^{4}\text{He}_a}, \]

where the subscript \( a \) denotes standard atmospheric conditions.

The three primary sources of helium in an aquifer are the atmosphere, the lithosphere, and the mantle, each of which has a distinct isotopic signature. Shallow groundwater tends to have an isotope helium ratio similar to the atmospheric value (\( R/R_a \approx 1 \)), as it is greatly influenced by precipitation that infiltrates the subsurface. Radiogenic production of
helium in the lithosphere due to the decay chains of uranium ($^{235,238}$U) and thorium ($^{232}$Th) present in crustal minerals has a production ratio generally less than 2% of the atmospheric ratio [Ozima & Podosek, 2002]. Therefore, crustal waters with significant residence times often exhibit $R/R_a \ll 1$. The highest ratios are found in tectonically or magmatically active regions where magma-derived gases with proportionately higher concentrations of $^3$He infiltrate crustal fluids. In such regions, groundwater exhibits isotope helium ratios ranging from about $1R_a$ to over $20R_a$ [Ballentine & Burnard, 2002], although $6R_a$ is a more common value [Dunai & Porcelli, 2002] for continental regions. It is important to note, however, that low $R/R_a$ ratios, sometimes significantly below $1R_a$, can still represent considerable magma-derived $^3$He contributions in regions where large amounts of radiogenic $^4$He production occur [Castro, 2004; Saar et al., 2005].
Chapter 3

Models

3.1 Conceptual Model

I examine a simple two-dimensional system (Figure 3.1), designed to be representative of a groundwater system in a region of high heat flow, such as the Basin & Range province in the western United States. Basal heat and helium fluxes create high temperature and $R/R_a$ values at the base of the models, while at the top, temperature and $R/R_a$ are consistent with near-surface groundwater values. No-flow boundary conditions for fluid, heat, and helium exist across the vertical boundaries of the model. All simulations run until steady-state is achieved. Multiphase flow and hydrodynamic dispersion are not considered. All properties of the porous medium, such as porosity, permeability, and pore-space tortuosity, are isotropic and homogeneous throughout unless otherwise stated.

A 10% slope in the pore-fluid pressure along the top of the model represents the slope of a water table, allowing groundwater recharge and discharge (Figure 3.1). Neumann boundary conditions for heat and helium are imposed at the base of the system, representing
contributions of these natural tracers from a cooling and degassing magma source. A heat flux of 90 mW/m$^2$ is imposed at the base of the model, an average value for heat flow in the Basin & Range province [López & Smith, 1995] and approximately equal to the mean global heat flux of 87 mW/m$^2$ [Ingebritsen & Sanford, 1998]. Basal helium flux is determined from estimates of $5 \times 10^9$ atoms m$^{-2}$s$^{-1} \approx 5 \times 10^{-17}$kg m$^{-2}$s$^{-1}$ by O’Nions & Oxburgh [1983], with $R/R_a=6$, for typical magma conditions. No fluid flow occurs across the base of the model. Dirichlet conditions for surface temperature and helium concentration along the recharge zone allow for heat and helium flux across the top boundary. Boundary conditions along the discharge zone vary according to property and the purposes of the simulation. Details regarding the boundary conditions used in a model are given in the appropriate sections.
I ignore variations in fluid density and viscosity due to variable temperature because of the modeling software used in this research, for reasons explained in Section 3.2. However, Rayleigh number analysis suggests that this is an acceptable simplification of the problem. The Rayleigh number, $Ra$, is calculated using the equation

$$Ra = \frac{\rho_0 \alpha g k L^2 J}{\mu k_m \lambda_m},$$

(3.1)

where $\rho_0$ is a reference fluid density (1000 kg/m$^3$), $\alpha$ is the thermal expansion coefficient of water ($3 \times 10^{-4}$ $1/\degree$C), $J$ is the basal heat flow, $\mu$ is the dynamic fluid viscosity ($10^{-3}$ kg/[m s]), $k_m$ is the mixed thermal diffusivity ($6.7 \times 10^{-7}$ m$^2$/s), and $\lambda_m$ is the mixed thermal conductivity (1.8 W/[m $\degree$C]). Substituting these values into the equation results in $Ra \approx 2.2$, a Rayleigh value well below the critical value ($Ra_c = 17.65$) for the onset of buoyancy-driven convection [Simms & Garven, 2004].

### 3.2 Numerical Methods

Simulations are performed using two software packages. The first is the geothermal reservoir simulator, TOUGH2 [Pruess et al., 1999]. I use the equation of state module for noble gases, EOSN [Shan & Pruess, 2003], modified to include temperature-dependent helium diffusion and radiogenic heat and $^4$He production [Andrews et al., 2007]. The second is the code, COMSOL Multiphysics. I use a combination of three physics modes available from the Earth Sciences Module: Darcy’s Law (esdl), Convection and Conduction in Porous Media (eshcc), and Solute Transport (esst).

TOUGH2 is ultimately the more appropriate code for simulating heat and helium transport in geothermal reservoirs. It is a robust multiphase, multi-component simulator.
that uses the integrated finite difference method for solving transport equations. TOUGH2 accounts for variable fluid density and viscosity due to variations in pressure and temperature. It is also capable of dealing with dual-porosity, anisotropy, and heterogeneity. Stark permeability contrasts do not generally pose a problem with this code.

The disadvantage to using TOUGH2 is the inflexibility for prescribing boundary conditions. A boundary element must have either a Dirichlet or a Neumann condition for all variables in the system. For example, the user cannot define a constant pressure condition and a heat flux condition in the same element. Either pressure and temperature must both remain constant, or both fluid and heat flux across the element must be defined. Further, if a fluid flux condition exists, the user must specify an enthalpy of the fluid. This implicitly defines the temperature of fluid in the element through the relationship between pressure and enthalpy. If the enthalpy of fluid entering or leaving the element is zero, there will be no heat transfer across the boundary at that element.

COMSOL Multiphysics, which utilizes the finite element method, has a broader interest base than TOUGH2. It encompasses a wide variety of physics-based modules that the user can mix and match from several disciplines to obtain the appropriate set of equations for her research interests. In order to make this possible, COMSOL offers great flexibility regarding how variables are coupled in a system. Because flexibility in defining boundary conditions is a large part of my thesis research, I have made extensive use of this software. The results presented here are all the product of simulations performed using COMSOL.

There are disadvantages to using COMSOL for modeling geothermal systems, however. While the option exists to incorporate fluid properties as a function of temperature,
numerical solutions become unstable for systems in which convective heat transfer is significant ($Pe \geq 1$). At present, COMSOL provides limited usefulness for geothermal reservoir analysis. Simulators such as TOUGH2 are vital for future research of this topic.
Chapter 4

Boundary Conditions

Appropriate application of boundary conditions in conceptual models is vital to the design of a valid numerical model. Therefore, I present here a brief discussion of the three primary methods of constraining boundaries in groundwater modeling. Each of the three categories is known by a multitude of names, which can create a great deal of confusion as to what each condition is and how it is used in building a conceptual model.

This is not meant to be an exhaustive discussion of the topic. A more in-depth understanding can be gained from groundwater modeling books such as those by Anderson & Woessner [1992] and Batu [2006]. Further details regarding implementation for various numerical methods and software packages can often be obtained from the appropriate user guides.

This chapter concludes with problems encountered modeling coupled heat and helium transport as it relates to conditions along boundaries and my attempt at providing a more elegant solution to the problem.
4.1 Conventional Boundary Conditions

4.1.1 The Dirichlet Condition

The Dirichlet boundary condition, also known as the Type I, First-Type, or Constant-Potential boundary condition, is expressed as

\[ \Psi = \psi(x, y, z, t), \]  

(4.1)

where \( \Psi \) is the value of some variable in the system, such as pressure, temperature, or concentration. Spatial and temporal coordinates are represented by \((x, y, z)\) and \(t\), respectively.

For this boundary condition, a constant value of pressure, temperature, or concentration is specified for the element or boundary at a point in space and time. This allows for both diffusive and advective flux of a tracer across the boundary.

The Dirichlet condition assumes the value of a variable is known within some reasonable error. As such, this is a good condition to use when field data are available for input into a conceptual model or when general approximations can be made. For example, one valid use of this condition includes defining pressure or hydraulic head when the location of the water table or height of the potentiometric surface is known. If infiltration from precipitation events is the primary source of groundwater recharge, then temperature and concentration of near-surface waters in the recharge zone will have signatures similar to meteoric water. Therefore, another valid application of the Dirichlet boundary condition involves the use of atmospheric conditions to define temperature or solute concentration at the recharge zone of a groundwater flow system. Depending on the length of time considered in a simulation, the modeler must determine what frequency of the signal is most appropriate. For example, steady-state simulations require only a low-frequency...
temperature estimate such as mean annual temperature, while studies that consider daily fluctuations in close proximity to the atmosphere must rely on high frequency hour-by-hour or minute-by-minute temperature variations.

This condition must be used with caution along discharge zones. Defining a constant atmospheric boundary condition imposes unrealistically high gradients near the surface at the discharge zone for systems with substantial advective fluid flow [Smith & Chapman, 1983]. It also prohibits the formation of springs that carry a signal different from the one defined in the condition.

4.1.2 The Neumann Condition

The Neumann condition is sometimes referred to as the Type II, Second-Type, or Constant [Diffusive] Flux boundary condition. It is expressed by the equation

$$d_n = -D_{e,c} \frac{\partial \Psi}{\partial n},$$

(4.2)

where $d$ is diffusive flux, $n$ denotes the direction normal to the boundary, and $D_{e,c}$ represents the effective diffusion coefficient of the variable of interest, $\Psi$.

The Neumann condition describes only diffusive flux across a boundary and relates nothing about advective tracer transport. This boundary condition is appropriate for defining no-fluid-flow boundaries. This is accomplished by setting the pressure or hydraulic head gradient equal to zero ($\frac{\partial P}{\partial n} = 0$ or $\frac{\partial h}{\partial n} = 0$), i.e., pressure or hydraulic head diffusion is zero, which results in no fluid flow across the boundary. Conversely, in order to define a particular fluid flux across the boundary, the pressure or hydraulic head gradient is made non-zero. Similarly, diffusive flux of a tracer is regulated by defining a gradient of the variable across the boundary. When diffusive flux of a tracer is made zero, the condition is
often referred to as a Convective Condition because convection is then the only way for the tracer to move across the boundary.

This condition does not assume a specific value for pressure, temperature, or concentration and, therefore, values of these variables along the boundary are allowed to change. However, the user must know something about movement of the tracer across the boundary. Diffusive flux is generally not simple to measure directly in a natural Earth system. However, flux values for a region can sometimes be inferred when sufficient data exists to obtain a ‘feel’ for what is going on in the system. For example, temperature measurements at several depths in a borehole may yield an estimate of heat flow. Estimates of flux can also be shared by (or borrowed from) similar systems as a first approximation; the use of 40 mW/m$^2$ for a stable sedimentary basin (Ingebritsen & Sanford, 1998) or 90 mW/m$^2$ for a volcanic region (e.g., López & Smith, 1995) for heat flow estimates are examples of this approach. The use of a conductive basal flux of heat or any other tracer bypasses the need to make assumptions of a maximum value for the system or assumptions of large-scale temperature, concentration, or transport patterns.

4.1.3 The Cauchy Condition

The Cauchy boundary condition is known by several aliases, including the Type III, Third-type, Head-dependent, Generalized Neumann, and Robin condition. With this condition, flux across a boundary is defined as

$$q_n = u_n \Psi - D_{e,\Psi} \frac{\partial \Psi}{\partial n},$$  \hspace{1cm} (4.3)

where $u_n$ is the seepage velocity across the boundary.

The interior elements of a model all technically have this third-type boundary
condition, as it is simply the advection-diffusion equation (Equation 2.1). Inside the model domain, the equation is easier to solve because a value at one node is constrained by the values at all adjacent nodes which, in turn, are constrained by the boundaries of the numerical model. The Cauchy condition is a function of pressure or hydraulic head through the velocity relation defined in Darcy’s Law,

\[ \phi u = -\frac{k}{\mu}(\nabla P + \rho g) = -K \nabla h, \]  

(4.4)

where \( \phi u \) is Darcy velocity, \( k \) is permeability, \( \mu \) is dynamic fluid viscosity, \( P \) is pore-fluid pressure, \( g \) is gravitational acceleration, and \( h = P/(\rho g) + z \) is hydraulic head. Therefore, the Cauchy boundary condition can only be applied to tracers (e.g., temperature or solute concentration) and not to pressure or hydraulic head.

To use this condition along a boundary, the user must specify both diffusive and advective flux of the tracer, which can be difficult. Therefore, conditions along a specific boundary must be strictly controlled or known extremely well.

Since this condition requires that the boundary be essentially micromanaged and because surface-air interactions are extremely complex and variable, this is generally an inappropriate condition for a natural boundary. Heat, solute, and water transfer are very complex and influenced by several factors, including fluid flux across the boundary, depth of the surrounding vadose zone, materials present in and below the vadose zone, extent and type of vegetation at the surface, wind velocity, atmospheric humidity, and air temperature. There are few, if any, viable generalized methods for this condition along a natural boundary of a groundwater reservoir.

There is an important difference between the Robin condition and the Cauchy condition. With Cauchy, both diffusion and advection are defined separately, while with
the Robin condition both diffusive and advective transport across the boundary are treated as a lump sum. In this context, the Convective condition (Section 4.1.2) is really a variation of the Cauchy condition; diffusive flux is defined as zero while advective flux is determined separately by the velocity of fluid across the boundary.

4.2 Boundary Conditions for Helium

Prior to submission of a manuscript, I performed numerous simulations of heat and helium transport in which Dirichlet atmospheric conditions prevailed for the entire surface of the model. A helpful reviewer pointed out that this was an incorrect surface constraint for helium in a groundwater discharge region. As discussed previously, utilizing a Dirichlet, constant atmospheric concentration condition imposes unrealistic constraints on the system. In real groundwater systems with sufficiently vigorous subsurface flow, the upflowing water is not able to equilibrate with the atmosphere and therefore has characteristics more indicative of the subsurface rather than the near-surface or atmosphere. For example, at hot springs, high convective heat flow prevents cooling of spring waters through conductive heat loss. Additionally, discharge zones typically have high solute concentrations resulting from prolonged residence times and the low capacity of most solutes to volatilize from solution and escape the groundwater system to the atmosphere.

The argument I had used to justify the original boundary condition was that the top boundary represented the upper groundwater in equilibrium with air-saturated water. I thought that if the top row of cells were removed, the result would be the same as if a convective flux boundary existed along the top. However, this is not the case. As can be seen in Figure 4.1, the difference between using Dirichlet and Neumann conditions for
helium along the discharge zone is significant. While helium concentrations in the bottom half of the system appear to be the same in both cases, signals near the surface are much greater when the Convective condition (Figure 4.1b) prevails along the discharge boundary.

According to Mazor [1997], significant amounts of noble gases do not diffuse from the water table due largely to the hydrostatic pressure within the saturated zone. Therefore, atmospheric values of helium concentration and $R/R_a$ are inappropriate in discharge regions, even if upflow is minimal. According to the reviewer, the correct surface boundary condition for any solute, helium included, along a discharge zone is the Convective condition such that diffusive flux is zero and the only way for helium to leave the system is as a component of the discharging fluid.

Figure 4.1: Normalized helium concentrations for the cases of (a) Dirichlet condition and (b) Convective (zero diffusive flux) condition along the discharge zone. The system shown here has $k = 10^{-16}$ m$^2$. Refer to Figure 3.1 for flow regime.
4.3 Boundary Conditions for Heat

Upon investigating the correct boundary conditions for helium, consideration of appropriate boundary conditions for heat is a natural transition. The most common boundary condition for heat along the surface is a Dirichlet condition. This is partly because of its simplicity but also because it is often a valid condition. Heat readily conducts out of the subsurface, resulting in atmospheric temperature conditions along many discharge boundaries. However, there are circumstances in which modelers have used a Convective (no diffusive flux) boundary condition for heat. In particular, the modeling of hot springs requires a non-Dirichlet condition, at least where springs are expected or known to occur. For example, Forster & Smith [1988a; 1988b; 1989] and López & Smith [1995; 1996] designate a fracture zone as the primary discharge region and define a convective condition for heat at the fracture. This allows temperature of the discharging spring to change with the transport regime and basal heat flow. The problem with this approach is that it is very restrictive in regards to the location and extent of high convective heat transfer and, therefore, the formation of hot springs.

The Hybrid Condition

The hybrid boundary condition, introduced here, is an attempt to address concerns about boundary conditions along a continuous surface exposed to variable advective fluid flow rates. The method allows temperature or solute concentration along the boundary to adjust for both atmospheric and subsurface conditions. It is designed to simplify the modeling of a single system or a series of systems that employ one geometry but have multiple permeability structures.
According to this condition, a potential, $\Psi$, such as temperature or concentration, along a boundary at a point $(i,n)$ is defined as

$$
\Psi_{i,n} = \begin{cases} 
\Psi_{atm}, & v_n \leq v_t \\
\Psi_{i,n-1}, & v_n > v_t,
\end{cases}
$$

(4.5)

where $v_t$ is a transition velocity, $v_n$ is the fluid velocity normal to the boundary at that location and $\Psi_{i,n-1}$ is the potential of the element or node directly below the boundary element. Elements with fluid velocities less than the transition velocity have a constant, atmospheric potential. Higher velocities result in a purely convective boundary condition, such that the gradient of the potential of interest is zero, prohibiting conduction across the boundary at that element.

In a COMSOL simulation of heat transfer, for example, the hybrid boundary condition is implemented by defining the temperature of the boundary segment as

$$
(v_{esdl} \leq vel_t) * Tatm + (v_{esdl} > vel_t) * T[x][y-d],
$$

(4.6)

where $v_{esdl}$ is the vertical fluid velocity, $vel_t$ is the transition velocity, $T[x][y-d]$ is the temperature at a depth, $d$, below the surface, and $x$ and $y$ denote the horizontal and vertical axes, respectively. As discussed in Section 3.2, this hybrid condition cannot be implemented in TOUGH2.

Deciding what $v_t$ ought to be is not a simple task. For the simulations presented here, the transition velocity is based on $Pe_{1D} = 1$ for a characteristic length of 1000 m, which is the depth of the model. Assuming a diffusivity for heat of $10^{-6}$ m$^2$/s, according to Equation 2.8, the transition velocity is $v_t = 10^{-9}$ m/s. As was stated previously, this condition assumes that there is a transition velocity above which conduction of heat can be ignored. For $Pe = 1$, conduction is still a significant part of heat transport. Therefore, a
Figure 4.2: Temperature (°C) patterns for systems with the following boundary conditions along the discharge zone: (a) Dirichlet (b) Hybrid (c) Convective (Neumann with zero diffusive flux).

much higher value of $Pe$ than the one used here seems necessary. However, because of the numerical instabilities encountered at moderate permeabilities with COMSOL (Section 3.2), I chose a low Peclet value to enable comparison of the differences encountered with the various boundary conditions.

The model presented in Figure 4.2 has a permeability of $k = 10^{-15}$ m$^2$, such that the two-dimensional Peclet number for heat is $Pe_{2D} = 10$. At this permeability, the effect of convective heat transport on temperature patterns is noticeable. For example, curved isotherms exist in Figure 4.2a; a conduction-dominated system would have parallel isotherms. When Dirichlet (atmospheric temperature) conditions are imposed along the discharge zone (Figure 4.2a), the system must adjust by creating a high temperature gradient near the surface (upper right corner).

Figure 4.2b shows the temperature pattern when the hybrid boundary condition is imposed along the surface. In this case, atmospheric temperatures extend into the discharge region near the center of the system. Further into the discharge zone, the higher fluid velocities force the switch between the Dirichlet and Convective boundary conditions, such
that elevated temperatures exist along the surface. For comparison, the system shown in Figure 4.2c has a strict Convective boundary condition, in which no conductive heat loss to the atmosphere occurs along the discharge zone.

In this example, when a convection condition is used for heat, whether in the form of the hybrid or convective boundary condition, elevated temperatures exist at the discharge zone. This is because heat is being focused and is unable to escape via conductive loss to the atmosphere. This does not mean that the hybrid condition is invalid. The chosen value for the transition velocity is rather low. A more appropriate value may prevent the concentration of heat at the discharge zone, allowing for more realistic temperatures for discharging springs.

The advantage to using the hybrid boundary condition lies in its flexibility. Defining a purely convective condition at a specific location may be appropriate when the location of a spring or springs is well known. However, for generalized systems like the one presented here, or when modeling fluctuations in the groundwater flow regime, the location of discharge zones and the extent of fluid flux discharging from the system cannot necessarily be known a priori. This hybridized method allows conditions along the surface to change according to the rate and pattern of fluid flow, a technique which is more representative of the physical processes occurring in a natural system.
Chapter 5

Results and Discussion

This chapter includes results and brief discussions of the primary factors influencing heat and helium signatures in groundwater systems. This chapter gives visual presentation to these processes and properties, demonstrating how each affects analysis of heat and helium measurement.

5.1 Dilution of Magmatic Helium Signals

Occurrence of decoupled heat and helium signals is often caused by the addition of radiogenic $^4$He, a by-product of uranium ($^{235,238}$U) and thorium ($^{232}$Th) decay. This crustal helium has a different isotopic signature and can obscure the magmatic helium component while leaving the heat component intact.

To investigate how the inclusion of radiogenic helium production affects isotope ratios, source terms for heat and $^4$He are added to each element. The radiogenic heat production rate is 0.8 $\mu$W/m$^3$, a value consistent with limestone, dolomite, sandstone, alluvium, and fault zones [Clauser & Villinger, 1990]. Production of $^4$He is estimated
Figure 5.1: Radiogenic production of $^4\text{He}$ has a large impact on helium isotope ratios. Contours of $R/R_a$ for a system with (a) no in-situ production of $^4\text{He}$ and (b) in-situ production rate of approximately $1.5 \times 10^{-13}$ kg/(m$^3$ yr) using the ratio $3.7 \times 10^{-8}$ (cm$^3$ $^4\text{He}$(STP))/J [Ballentine & Burnard, 2002]. Radiogenic production of $^3\text{He}$ is ignored, as crustal production of this isotope is approximately eight orders of magnitude less than the production of $^4\text{He}$ [Ozima & Podosek, 2002].

Figure 5.1 shows how ignoring and considering radiogenic production of $^4\text{He}$ impacts helium isotope ratios. For the case where radiogenic production is ignored (Figure 5.1a), the maximum value is the basal influx value, $R/R_a \approx 6$. However, when radiogenesis is considered, values of $R/R_a$ are drastically reduced throughout the system (Figure 5.1b). In this case, the maximum value in the system is equal to the value $R/R_a \approx 1$ for air-saturated water at the recharge zone.

5.2 Effect of Diffusion on Heat and Helium Signals

A significant cause of decoupled heat and helium signatures is the varied diffusion coefficients of heat and helium. The diffusion coefficient of helium is approximately two
Figure 5.2: Decoupling of temperature and helium signals due to their different rates of diffusion.

orders of magnitude lower than that of heat. Because of this, helium transport in low-permeability materials may be dominated by advective processes while the dominant mode of heat transport is conduction [Manning & Ingebritsen, 1999; Bickle & McKenzie, 1987; Torgersen, 1993; Clauser et al., 2002]. For permeabilities less than $10^{-16}$ m$^2$, heat flow is often dominated by conduction, while helium transport is primarily diffusive [Manning & Ingebritsen, 1999].

Figure 5.2 shows temperature and helium values for a system with a permeability of $k = 10^{-18}$ m$^2$. The parallel isotherms in Figure 5.2a show that heat transport is conduction-dominated. In contrast, the curved concentration contours show that helium transport is dominated by fluid flow (Figure 5.2b). Consequently, the discharge zone has a relatively high magmatic helium signature but a temperature signal consistent with atmospheric conditions.
Figure 5.3: (a) Concentration of helium in a homogeneous system. (b) Helium concentration in a heterogeneous system. The dotted lines at z=400 m and 600 m enclose a low-permeability layer that acts as an entrapment zone for helium.

5.3 Helium Entrapment

While conductive heat transport is possible through layers effectively impermeable to fluid flow, helium is prone to entrapment, relying on seismic activity to create and maintain permeable conduits for transport [Torgersen & O’Donnell, 1991; Hilton, 1996, Kulongoski et al., 2005]. Figure 5.3 shows the effect of a low-permeability layer on helium signals. In this case, the primary material has $k = 10^{-18} \text{m}^2$, while the low-permeability layer has $k = 10^{-21} \text{m}^2$. Elevated helium concentration occurs within and below the entrapment zone. Above this layer, helium concentrations are greatly diminished by two orders of magnitude. This is because the low-permeability layer hinders infiltration of the basal (magmatic) helium to the surface aquifer.
5.4 Temperature-Dependent Helium Diffusion

Still of interest to note is the effect of considering temperature-dependent diffusion coefficients on helium concentration-depth profiles.

Figure 5.4 shows the difference between using temperature-dependent and constant diffusion coefficients of helium. Following the method of Bredehoeft & Papadopulos [1965] Bickle & McKenzie [1987], I consider variation of helium concentration with depth in a low-permeability layer bounded above and below by higher permeability aquifer units (refer to Figure 5.3b). I use the concentration profiles for a transect [x=850 m] of the low-k layer presented in that figure.

When including temperature-dependent helium diffusion coefficients in a system with a variation in temperature, any curvature in the concentration-depth profile is accentuated by the variable diffusion coefficients. Therefore, while attributing the curvature of a concentration-depth profile entirely to advective transport is valid for a solute with an essentially constant diffusion coefficient, this is not necessarily the case for helium.

For the scenario modeled here, with a temperature range of 11°C and a depth of 200 m, the difference between using constant and temperature-dependent diffusion coefficients results in an approximately 1.5% variation in helium concentration (Figure 5.4). This is much less than the current average measurement error of 6.5% [Saar et al., 2005]. While inclusion of temperature-dependent helium diffusion may not be necessary at present, it may become more important in the future as measurement errors decrease.
Figure 5.4: The temperature-dependence of helium diffusion results in non-linear concentration-depth profiles. (a) Concentration of $^4$He, normalized to the maximum value, for the cases of a constant $D_{\text{He}} = \text{constant}$ and temperature-dependent $D_{\text{He}} = f(T)$ diffusion coefficients. [Inset: Temperature over the same depth scale.] (b) A linear profile is subtracted from both helium profiles to show the extent of influence from advective helium transport (c) Difference in the two profiles. The maximum difference is approximately 1.5%.
Chapter 6

Avenues for Future Research

Future graduate students have the opportunity to expand our understanding of heat and helium transport dynamics. In this chapter, I present three ideas for potential graduate research. The first project idea is experimentally intensive, while the last two involve transient numerical modeling of geothermal systems.

6.1 Helium Diffusion

The aim of this potential research project is to fill the gap in experimental data involving coefficients of helium diffusion. The first phase is to measure the diffusion coefficients of $^3$He and $^4$He to determine whether there is indeed a measurable and significant difference between the rates of diffusion of the two isotopes (Section 2.3). The second phase concerns measurements of the diffusion coefficient of helium and possibly other noble gases at higher temperatures than the maximum of 35°C performed by Jähne et al. [1987].

If time permits, this project could also involve some numerical modeling. For large temperature ranges, the modified version of EOSN [Andrews et al., 2007] could be used to
investigate different scenarios of $^3$He and $^4$He transport.

### 6.2 Helium and Seismicity

The first process to consider by way of numerical modeling involves the instantaneous release of helium from low-permeability layers after seismic events. Torgersen & O’Donnell [1991] performed a 1-D analysis of helium and argon release due to fracturing. However, there do not appear to be any larger-scale investigations.

Modeling release of heat and helium due to seismic events involves a two-step process. The first step is to model a steady-state system in which helium is trapped in a lower permeability basement layer. Second, the creation of a fracture zone by changing the permeability of a narrow region allows for a view of the propagation of the magmatic helium component through the basin. This can be followed by a comparison of the movement and pattern of helium to the perturbations of heat transport paths and temperature patterns. Such a study would provide helpful information regarding the effect of seismicity on heat and helium signals.

What is particularly problematic with my helium transport models, is that they are all steady-state simulations. With system-wide radiogenic helium production, when steady-state conditions prevail, magmatic signals are completely obscured and the propagation of a high-$R$ mantle signature through the system cannot be shown. In order to show that, transient simulations should be considered.
6.3 Multiphase Flow

The second process of interest for the numerical modeler is multiphase flow, and phase separation in particular. The questions to answer are (1) what happens to heat versus helium signals during phase separation events that might cause decoupling of the two signatures and (2) whether isotopic fractionation is significant in measurement and analysis of helium isotope ratios.

Due to the low solubility of helium in water, phase separation in multiphase systems can lead to enrichment of helium in the vapor phase (depletion in the liquid phase), while allowing for isotopic fractionation, such that the residual liquid is enriched in the heavier isotope compared to the parent liquid [Truesdell et al., 1977].

Consider the separation of $^3$He and $^4$He during phase separation and also the fractionation of helium and heat during phase separation. Isotopic fractionation is not significant when the process is batch equilibrium degassing but may be significant during fractional (Rayleigh) equilibrium degassing [Hilton, 1996]. At Long Valley Caldera, Hilton [1996] determined that the He-C isotopic system is controlled primarily by batch equilibrium degassing and radiogenic helium enrichment. Research of fractionation of other chemical species, such as C, Cl, O, and H, has also been conducted [e.g., Truesdell et al., 1977; Clark & Phillips, 2000; Torgersen et al., 2004; Inguaggiato et al., 2005].

While phase separation of various chemical species, including helium, has been considered by others, there are no numerical models of the process in which helium signatures are specifically modeled. Consequently, there have been no numerical models that couple heat and helium during phase (steam) separation. This may be an important part in understanding decoupling of heat and helium signals. The study of the effects of phase
separation on heat and helium signals is a necessary addition to a well rounded body of knowledge concerning heat and helium in geothermal reservoirs, since many geothermal reservoirs, particularly the most lively ones, exhibit steam separation at some level.
Bibliography


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Appendix A

Helium Data

The question of heat and helium decoupling started with a paper by Clauser et al. [2002] about decoupling of heat and helium signals in the Rhine Graben of southwestern Germany. They make extensive use of a geochemical data set compiled by Griesshaber et al. [1992] for waters in and around the graben, including three crowning volcanic systems, the Vogelsberg, Kaiserstuhl, and Eifel. In the graben, the highest mantle-helium signatures are over the volcanic regions, while the highest temperatures are found in upwelling zones in the graben.

I have a few problems with their comparison of data. The data for heat, helium, and other chemical species were taken at different depths, in different phases, and at different times. Additionally, the thermal data is an entirely different data set.

The comparison of chemical species across phases can be problematic, in that there can be significant differentiation of geochemical tracers during phase separation. The fact that samples across phases were compared is not necessarily detrimental to their findings, however. If the system has attained chemical equilibrium (as discussed in Hilton [1996]
for Long Valley Caldera), comparisons can be made across phases, even in a tectonically dynamic system. Hilton [1996] characterized his samples by comparing the ratio CO$_2$/\(^3\)He. However, Griesshaber et al. [1992] never made the argument or analyzed the data to show that the phases are in equilibrium, so it is uncertain whether the comparisons they make are valid.

In that same publication by Hilton about Long Valley Caldera, the effect of helium release after seismic events is also given considerable discussion. Like Long Valley Caldera, the Rhine Graben is seismically active, and the creation of fractures during seismic events could conceivably cause ‘pulses’ of high helium concentration through the system. Because samples were taken over such a large range of times, I am not sure that the comparison is valid. There is no mention that anyone was able to determine whether seismicity played a part in the geochemical signal variations or not.

The next problem, is that in the paper by Clauser et al. [2002], they use temperature data from a depth of 1 km, while all of the helium and other geochemical data come from depths of a few hundred meters or less. Therefore, it is uncertain whether their claim that the signals are decoupled is a valid one. They are not comparing the temperature of a sample and the geochemical signature of that same sample. They are comparing near-surface with deep subsurface data.

Even with these complications, there is still something to be gained from the data used by Clauser et al. [2002]. There is enough self-consistency among the thermodynamic and geochemical data that one could safely say that the data sets as a whole are representative of regional-scale trends. Because of the issues listed above, the data sets do not lend themselves to direct comparison or detailed scrutiny. However, they should be sufficient to
discuss the problem at large.
Appendix B

The ’Borrowed’ Boundary Condition

If the Dirichlet, Neumann, and Convection boundary conditions are not necessarily appropriate for discharge boundaries, how can heat and fluid transport be simulated in a series of systems with a large variation in permeability? Inspiration came from a paper about a CO$_2$ transport boundary developed by Oldenburg & Unger [2004] for TOUGH2. In this paper, they present the results of simulations in which they monitor the seepage of CO$_2$ from a sequestration site. CO$_2$ slowly escapes through caprock, travels through the water table and vadose zone out into the atmosphere. Because they were able to develop a way to allow solute concentration to change along the surface of a boundary while holding pressure constant, this seemed like a promising approach to the heat boundary condition issue of my models. However, the transition from one model to the next is not a simple—or possible—one.

In the paper presented by Oldenburg & Unger [2004], only dispersive and diffusive
loss through the vadose zone into the atmosphere are accounted for. Water flow through an aquifer with recharge and discharge zones is not considered. The only fluid to flow through their model is air, which enters and exits through constant pressure boundaries along the uppermost sides of the model (Figure B.1).

For their method to work on my model, the target model, I would need to find a way for water to enter and exit the aquifer. This could be done by adding an extra row of cells behind the surface boundary. Water flow could be diverted to those cells, just as spring discharge is diverted to a stream. However, in order for the ’stream’ boundary elements to allow water to pass through them and out of the model, a constant pressure condition must be prescribed. In TOUGH2, this also means a constant temperature condition must be defined, which was the initial problem.

Essentially, the problem with adopting this model to my target is a matter of scale. Oldenburg & Unger [2004] model a region small enough that regional groundwater
flow can be ignored. At this scale along the vadose zone, the atmosphere is the primary control on subsurface signals. In the target model, a combination of variation in pore-fluid pressure along the water table, material permeability, and basal heat flow are the primary determiners of fluid and tracer transport. Therefore, a transformation between models and scales in this instance is not feasible.
Appendix C

Sample TOUGH2 Input File

Although I do not provide results of simulations performed using TOUGH2 in this thesis, TOUGH2 will undoubtedly become a prominent fixture in coupled heat and helium transport modeling. It will be particularly useful for modeling heterogeneous and multiphase systems. Therefore, I provide a sample of a TOUGH2 input file for using our modified version of the noble gas module, EOSN [Andrews et al., 2007]. The primary elements of this file to note are as follows.

The template for creating the file MESH for the system is included at the bottom of the input file (data block MESHMAKER). The mesh that is generated from this template provides for a row of 'dummy' elements along the surface of the model. This is necessary for proper implementation of Dirichlet conditions. The final MESH file is not included here, as it is quite large. In order to create dummy elements, the user must modify the mesh file as described in the TOUGH2 User’s Guide [Pruess et al., 1999].

In the ROCKS data block, there are two materials listed. The first is the primary material for this model, nicknamed 'BRIAN'. This is a standard format for material input,
except for the second line that provides a flag for radiogenic heat and helium production for that material. Full details on how to use this feature can be found in Andrews et al. [2007]. Also present in this data block is an extra material named 'DUMMY' that has all the same transport properties as 'BRIAN', but radiogenic heat and $^4$He are not produced in this material.

Data block 'DIFFU' is present with flags of 1.0 to signal to TOUGH2 to use temperature-dependent helium diffusion coefficients with a scaling factor of 1.0. More information regarding this option is also available in Andrews et al. [2007].

The GENER block is used to input basal heat and helium flux into all 100 elements at the base of the system.
Figure C.1: Sample TOUGH2 input file for use with the modified EOSN module, as discussed in Andrews et al. [2007].

```
TOUGH2 INPUT FILE - basic input file,
ROCKS----1----2----3----4----5----6----7----8
   BRIAN  1 2500.  1.E-17  1.E-17  1.E-17  2.0  1000.
   DUMMY  2500.  1.E-17  1.E-17  1.E-17  2.0  1000.
   MULTI----1----2----3----4----5----6----7----8
   5 6 2 8
   START----1----2----3----4----5----6----7----8
   123456789*123456789*1234
   PARAM----1----2----3----4----5----6----7----8
   2 500 500000000 0001 03 000 0
   1.E-06  1.0
   1.0000E5  0.0  0.0  0.0  0.0  0.0  0.0  0.0
   DIFFU----1----2----3----4----5----6----7----8
   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
   1.05e-8  1.05e-8  1.0
   1.05e-8  1.05e-8  1.0
   1.e-9  1.e-9
   NOBLE----1----2----3----4----5----6----7----8
   He 3 0 0
   He 4 0 0
   GENER----1----2----3----4----5----6----7----8
   CUI 1 Q 1 98 1 1 HEAT  9.0
   CUI 1 H3 1 98 1 1 COM3 2.995E-20
   CUI 1 H4 1 98 1 1 COM4  4.8E-15
   FU1 1 Q 1  HEAT  9.0
   FU1 1 H3 1 COM3 2.995E-20
   FU1 1 H4 1 COM4  4.8E-15
ENDCY
MESHEMKER1----1----2----3----4----5----6----7----8
   XYZ
   0.
   NX 100 10.
   NY  1 10.
   NZ 101  0.
ENDFI
```