Numerical simulation of aggradation and downstream fining

Simulation numérique d’évolution du lit et de l’abrasion des matériaux vers l’aval

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ABSTRACT
Rivers typically exhibit a tendency for grain size to become finer in the downstream direction. Data for a set of large-scale experiments on the aggradation of heterogeneous gravel have recently become available. These experiments show substantial downstream fining over several tens of meters. Here a decoupled numerical model for bed aggradation and downstream fining is developed in an attempt to test an existing gravel transport model against the experimental data. Generally good agreement is found between the predictions and the observations in the absence of all but trivial adjustments to the gravel transport model. The same transport relation does not perform as well for a corresponding case of uniform sediment. In all of the experiments the Froude number was close to unity, a condition which would suggest that a decoupled model might break down. Coupled and decoupled models for uniform sediment are thus compared for a case with Froude number very close to unity. They are also compared for cases in which the upstream water discharge, sediment feed rate and downstream water surface elevation vary strongly. The surprisingly good agreement between the two models suggests that concerns in the literature about the use of decoupled models may have been overstated.

RÉSUMÉ
Dans les rivières on a tendance à constater que les matériaux solides sont de plus en plus fins quand on se déplace vers l’aval. Des données expérimentales provenant d’essais à grande échelle sur des lits de graviers non homogènes sont disponibles depuis peu. Ces essais montrent une diminution sensible de la taille des matériaux sur plusieurs dizaines de mètres. La présente étude montre le développement d’un modèle numérique découpé pour l’évolution du lit et l’abrasion des matériaux vers l’aval, ce qui constitue une tentative de comparaison d’un modèle existant de transport de graviers avec des données expérimentales. Il a été trouvé en règle générale un assez bon accord entre les prédictions et les observations en l’absence de calage effectif au modèle de transport de graviers. La même relation de transport ne donne pas d’aussi bons résultats pour un cas de sédiments uniformes. Dans tous les essais le nombre de Froude était proche de l’unité, condition qui pourrait suggérer qu’un modèle découpé puisse être en défaut. Des modèles couplés et découpés pour un sédiment uniforme sont alors comparés dans ce cas de nombre de Froude très proche de l’unité. Ils sont aussi comparés dans des cas pour lesquels le débit liquide amont, le débit solide et la condition aval de

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niveau d’eau varient dans de larges limites. La bonne concordance observée avec surprise pour les deux modèles suggère que l’intérêt affiché dans la littérature en faveur des modèles découplés pourrait être exagéré.

1 Introduction

Speculation as regards the tendency for characteristic grain size to become finer in the downstream direction in mountain rivers has focused on two mechanisms. The first of these is clast abrasion or wear during transport (e.g. Sternberg, 1875; Mikos; 1993, Brewer and Lewin, 1993; Kodama, 1994), perhaps aided by the development of a weathering rind during periods of prolonged storage (Bradley, 1970). The second is the preferential transport of finer grains (e.g. Bradley et al., 1972; Paola et al., 1992b; and Paola and Seal, 1995). Field evidence suggests that both mechanisms can play a role in downstream fining, depending upon the setting. For example, Kodama (1994) and Mikos (1993) cite cases in geologically young environments where abrasion appears to dominate. Hoey and Ferguson (1994) have documented downstream fining in a geologically old environment where the clasts are sufficiently durable to exclude abrasion. Seal and Paola (1995) reported strong downstream fining in a geologically young environment, but over a rapidly aggrading reach that is too short for significant abrasion.

Quantitative field studies of the phenomenon are rendered difficult by a host of site-specific factors such as hydrologic regime, tributaries bringing in clasts of different lithologies, base level variation and the scale over which fining is manifested. This scale can vary from several kilometers to several hundred kilometers (Shaw and Kellerhals, 1982). Researchers have thus resorted to techniques which allow for isolating the effects of various parameters. Several numerical models of downstream fining have recently been developed (e.g. Parker, 1991a,b; Paola et al., 1992a; Van Niekerk et al., 1992; Vogel et al. 1992; Bezzola, 1992; Mikos, 1993; Hoey and Ferguson, 1994; and Paola and Seal, 1995). Most of these consider only selective transport, but the models of Parker (1991a,b) and Mikos (1993) also include abrasion.

Relatively little effort has been expended on experimental investigations of downstream fining. Abrasion does not lend itself to flume investigation, because the minimum distances over which its effect may be manifested are typically orders of magnitude larger than flumes. In addition, the first major attempt to study downstream change in grain size in a large flume yielded in the counter-intuitive result of downstream coarsening (Kodama et al., 1992). The reason for this result may be associated with their choice of grain size distribution. There is evidence that in certain environments pebble gravel may be preferentially transported over sand (Straub, 1935; Kellerhals, 1967). Using the experiment of Kodama et al. (1992) and the numerical model of Parker (1991a,b) as learning tools, a multi-university research team designed and performed a series of six large-scale experiments at St. Anthony Falls Laboratory (SAFL) on downstream fining (Paola et al., 1992b; Seal et al., 1995a, Toro-Escobar et al., 1996a). The experiments, which successfully manifested downstream fining, are outlined in more detail below.

The present paper is devoted to the development of a numerical model of downstream fining, and its verification using three of the SAFL experiments. The model explicitly includes the operative mechanisms and relies minimally on the fitting of coefficients. The part of the model relating to the transport of heterogeneous gravel and bed resistance was adopted directly from the surface-based bedload transport relation of Parker (1990a,b).

The point of the paper is not, however, the adaptation of an existing formulation to a somewhat different case. The configuration of the experiments at SAFL, although simple enough, was rather more complicated than the condition of quasi-normal flow assumed in Parker (1991a,b), and thus
presented some interesting numerical problems. The nature of these problems can be better understood after an overview of the experiments.

2 Overview of the experiments

Six experiments were performed in a channel at SAFL with a depth of 1.83 m, a width of 2.74 m and a length of 60 m. Runs 4 and 5 were conducted using the full width of the channel; for Runs 1, 2, 3 and 6 a narrower width of 0.305 m was used. The variation in width was intended to test the hypothesis that local patchiness in grain size distribution due to bar formation or braiding promotes downstream fining by selective transport (Paola and Seal, 1995). The width/depth ratio of the narrow runs was, however, sufficiently low to suppress the formation of bar patterns (Colombini et al., 1987).

The runs chosen for testing the numerical analysis presented here are Runs 1, 2 and 3. Full documentation can be found in Seal et al. (1995a) and Seal (1994); Seal et al. (1995b) gives a complete listing of the data. Water discharge $Q_w$, sediment feed rate $G_f$ and tailgate elevation $E_{th}$ were all kept constant during the run. The feed material was poorly sorted and weakly bimodal, with a mild deficiency in the pea gravel range (Fig. 1); geometric mean size $D_{50}$ was 4.63 mm, geometric standard deviation $\sigma_{f50}$ was 5.57 and specific gravity was near 2.65. The material was durable and thus not subject to noticeable abrasion by transport in the channel. The initial condition was that of a bare bottom containing flowing water that was ponded at the downstream end by a weir. Upon commencement of sediment feed, a mildly concave aggradational wedge ending in a front prograded downstream (Fig. 2).

![Grain Size Psi](image)

**Fig. 1.** Plot of the original and adjusted grain size distribution of the feed sediment. The adjusted grain size distribution used in the model excludes all grains finer than 2 mm and in addition has been modified to account for the fact that the small deposit upstream of the feed point in Fig. 2 was biased towards coarser grains.

Insofar as the goal was to use aggradation to drive downstream sorting, the runs were not continued to equilibrium, but rather halted when the front had arrived at a point between 35 and 40 m downstream of the feed point. The sediment discharge $G_s$ of Runs 2 and 3 was half that of the corresponding previous run. Some statistics about the runs are given in the Table; $L_f$ denotes the final deposit length. In all the runs slope decreased and depth increased downstream to the front; e.g. in
Run 2, slope $S$ decreased from about 0.018 to about 0.015; and depth increased from 0.15 m to 0.16 m from the feed point to the front. Upstream of the front the Froude number $F_r$ was everywhere only modestly above unity. A modest rise in water surface just beyond the front was interpreted to be an undular hydraulic jump (Paola et al., 1992b). A thorough sampling of surface and substrate material at the end of each run allowed for a complete temporal and spatial characterization of the sorting pattern (Seal et al., 1995a).

![Diagram](image)

Fig. 2. Schematic diagram of the configuration of SAFL Runs 1~3.

3 Numerical modeling of river aggradation and degradation

Most numerical models of aggradation and degradation describe the flow using the one-dimensional St. Venant shallow water equations. The sediment transport rate is specified as a function of flow parameters and bed evolution is computed by means of the Exner equation of sediment continuity. The equations of conservation of water momentum, water mass and sediment mass can be written as

\[
\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left( \frac{1}{2} u^2 + gh + g\eta \right) = \frac{-u_s^2}{h}
\]  

(1)

\[
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) = 0
\]  

(2)

\[
\frac{\partial \eta}{\partial t} + \frac{1}{1 - \lambda_p} \frac{\partial q}{\partial x} = 0
\]  

(3)

Here (3) pertains to uniform sediment; $\eta$ denotes bed surface elevation, $q$ denotes the volume sediment transport rate per unit width, $h$ and $u$ denote flow depth and velocity, $u_s$ denotes friction velocity, $x$ denotes the streamwise coordinate, $t$ denotes time, $g$ denotes the acceleration of gravity and $\lambda_p$ denotes bed porosity.
Most analyses also employ the quasi-steady approximation (e.g. De Vries, 1965), according to which the flow equations are taken as steady when the characteristic celerity of bed perturbations is small compared to that of water surface perturbations. That is, (1) and (2) are approximated to

\[
\frac{\partial}{\partial x} \left( \frac{1}{2} u^2 + gh + g\eta \right) = \frac{u^2}{h} \tag{4}
\]

\[u h = q_w \tag{5}\]

where \( q_w \) denotes water discharge per unit width. This procedure results in a considerable numerical simplification in that the flow equations are decoupled in time from the Exner equation. This simplification comes at a price. For example, when analyzing the response of a bed to rapidly changing boundary conditions, the time scale of interest concerning bed evolution may no longer be long compared to that of the flow, and the quasi-steady assumption may fail (e.g. Lyn, 1987). Another limitation occurs in the vicinity of the critical Froude number, where the water surface and bed celerities approach equality (e.g. Sloff, 1993), and the physical basis for decoupling becomes suspect.

Most numerical models of aggradation and degradation due to the differential transport of heterogeneous sediment are decoupled models. The treatments of Ribberink (1987), Chen (1987), Li et al. (1988), and US Army Corps of Engineers (1993) all rely on a backwater formulation for the flow, and are thus inherently limited to subcritical flow. The models of Rahuel et al. (1989) and Holly and Rahuel (1990a,b) are fully coupled, and thus should be able to treat supercritical flow conditions as long as the correct boundary conditions are imposed. Most of these models, e.g. those of Li et al. (1988) and US Army Corps of Engineers (1993), assume quasi-uniform flow whenever supercritical flow is encountered. This same assumption of quasi-uniform flow, according to which (4) further approximates to

\[u^2 = -gh \frac{\partial \eta}{\partial x} = ghS \tag{6}\]

has been used in most numerical models that specifically address downstream fining, e.g. Diegaard (1980) and Parker (1991a,b). The approximation may likely be valid over the better part of long river reaches that manifest downstream fining, and in addition conveniently skirts the issue of near- or supercritical flow. It cannot, however, be used in a numerical simulation of the SAFL runs, where the tailgate of Fig. 2 plays an important role in establishing the flow pattern.

The implication is that a fully coupled model may be required in order to treat near-critical flow and supercritical flow. Examples of such models are that of Lai (1991), which uses the multimode characteristics method, and that of Bhallamudi and Chaudhry (1991) which addresses the phenomenon of knickpoint migration. Both these models are for uniform sediment. Armanini (1989) has offered a fully coupled formulation for mixtures. Both coupled and decoupled models were developed for the present analysis. The coupled model applies only to uniform sediment. It is capable of treating subcritical, near-critical and supercritical flows, as well as an imposed rapidly changing hydrograph as a boundary condition. It is similar to
the model of Bhallamudi and Chaudhry (1991), except that it also allows the imposition of super-critical upstream boundary conditions. The decoupled model, which is a second order iterative scheme, applies to both uniform sediment and sediment mixtures. In addition, it is capable of treating the same range of cases as the coupled model, including near-critical flows and rapidly changing hydrographs, conditions under which a decoupled model might be expected to break down. There is little difference between the predictions of the two models. The decoupled model is inherently less stable than the coupled model in the case of Froude numbers very close to unity, and thus may need a larger space step or special treatment of time steps. At least over the range tested, however, the limitations to the decoupled model do not appear to be as severe as expected based on earlier research.

4 Formulation for sediment mixtures

Because the present treatment considers sediment mixtures, a language is needed for treating grain size distributions. The appropriate grain size is logarithmic, as outlined in Parker (1992). The customary choice is the $\phi$ scale;

$$D = 2^\phi$$ (7)

where $D$ denotes grain size in mm. This scale is counterintuitive in that increasing $\phi$ corresponds to decreasing grain size. Parker and Andrews (1985) and Paola and Seal (1995) have thus introduced the $\psi$ scale;

$$D = 2^\psi$$ (8)

which is used here. The range of relevant grain sizes is discretized into $N$ ranges, each centered on the values $\psi_1, \psi_2, \ldots, \psi_N$ in ascending order. The mass content fraction of the $j$-th grain size range in the surface, or active layer is denoted as $F_j$, where in general $F_j = F_j(x, t)$. The corresponding fractions for the bedload and sediment feed are $p_j(x, t)$ and $p_{fj}$. Note that by definition $F_j, p_j$ and $p_{fj}$ must sum to unity. Corresponding arithmetic mean grain sizes and standard deviations can be defined in terms of moments;

$$\overline{\psi}_s = \sum_{j=1}^{N} \psi_j F_j, \quad \overline{\psi}_l = \sum_{j=1}^{N} \psi_j p_j, \quad \overline{\psi}_f = \sum_{j=1}^{N} \psi_j p_{fj}$$ (9a,b,c)

$$\sigma^2_{\psi_s} = \sum_{j=1}^{N} (\psi_j - \overline{\psi}_s)^2 F_j, \quad \sigma^2_{\psi_l} = \sum_{j=1}^{N} (\psi_j - \overline{\psi}_l)^2 p_j, \quad \sigma^2_{\psi_f} = \sum_{j=1}^{N} (\psi_j - \overline{\psi}_f)^2 p_{fj}$$ (10a,b,c)

The geometric mean grain size and standard deviation of, for example, the surface layer are then given as

$$D_{sg} = 2^{\overline{\psi}_s}, \quad \sigma_{sg} = 2^{\sigma_{\psi_s}}$$ (11a,b)
The basis for the treatment of sediment conservation used here is a three-layer model, including a bedload layer, a surface, or active layer and a substrate layer. The use of less than three layers precludes the proper description of downstream change in grain size. It may be possible to obtain more detail with the use of models containing more layers (e.g. Di Silvio, 1992, Sieben, 1994). The Exner equation for uniform material is replaced by the form due to Parker (1991a) for mixtures, which derives from Hirano (1971); similar forms have been used by other authors (e.g. Bettess and White, 1981). For convenience of expression it is here broken into two parts;

\[
\frac{\partial \eta}{\partial t} + \frac{1}{1 - \lambda_p} \frac{\partial q_T}{\partial x} = 0 \quad (12a)
\]

\[
\frac{\partial}{\partial t} (L_\eta F_j) + \frac{1}{1 - \lambda_p} \left( \frac{\partial}{\partial x} (q_T p_j) - f_{i,j} \frac{\partial q_T}{\partial x} \right) = f_{i,j} \frac{\partial L_\eta}{\partial t}, \quad j = 1, 2, \ldots N \quad (12b)
\]

Here \( q_T \) denotes the total volume bedload transport rate per unit width summed over all sizes, \( L_\eta \) denotes the thickness of the surface (active) layer, and \( f_{i,j} \) denotes the mass content fraction in the \( j \)-th size range of the material at the interface of the surface and substrate layers. These last fractions characterize the way in which sediment is exchanged between the surface and substrate as bed elevation varies.

In the case of degradation it has been recognized since the work of Hirano (1971) that the surface layer mines the substrate as the bed degrades. That is, where \( f_{i,j}(x,z) \) denotes the mass content fraction of the substrate at point \((x,z)\) and \(z\) denotes a vertical coordinate,

\[
f_{i,j} = f_{i,j}|_{z = \eta - L_\eta} \quad (13)
\]

The case of aggradation, during which the exchange is from surface layer to substrate, however, is less clear. Parker (1991b) suggested that the surface layer acts as a kind of filter, such that some weighted combination of bedload and surface material is transferred to the substrate as the bed aggrades. A specific functional form for this was not, however, proposed. It was necessary to introduce an element of circularity to determine a functional form for the present analysis. In the companion paper by Toro-Escobar et al. (1996b), the sediment continuity equations (12a,b) and the measured surface and substrate size distributions of Run 3 were used to back-calculate the following approximate functional form for the exchange fractions \( f_{i,j} \):

\[
f_{i,j} = \chi p_j + (1 - \chi) F_j \quad (14)
\]

where \( \chi = 0.7 \). The above equation indicates that the substrate should be somewhat coarser than the bedload, but should be closer to the bedload than the surface layer (see e.g. Lisle and Madej, 1989; Lisle, 1995). Here (14) is adopted for the present numerical study. The use of an empirical equation determined from a data set to test the performance of a numerical model applied to the same data set is not as circular as might appear, because (14) was determined using data only from Run 3, while the numerical model was also tested against Runs 1 and 2. It is cautioned that although (14) is found to be reasonable for Runs 1 to 3, its general applicability remains untested.
Here the thickness of the active layer $L_a$ is set equal to a grain size loosely corresponding to the surface size $D_{s0}$. The grain size one standard deviation coarser than $D_{s0}$ is given by $D_s \sigma_{sg} = D_{s0} 2^{\alpha_{sg}}$. If the grain size distribution were perfectly log-normal then this size would correspond to $D_{s0}$, and $D_{s0}$ would be given by $D_s 2^{1.28 \alpha_{sg}}$. With this in mind it is assumed that

$$L_a = D_s 2^{1.28 \alpha_{sg}}$$  \hspace{1cm} (15)

This assumption is only slightly different from that used in Parker (1991a,b). A Keulegan-type relation is used to characterize bed resistance;

$$\frac{u}{u_s} = 2.5 \ln \left( \frac{11h}{k_s} \right)$$  \hspace{1cm} (16)

where the roughness height $k_s$ is set equal to $2L_a$. The relation gives results that are very similar to the Manning-Strickler relation used in Parker (1991a,b).

The formulation for bedload transport used here is the surface-based relation of Parker (1990a,b). It allows for the computation of the total bedload transport rate $q_T$ and the bedload size fractions $p_j$ at a point from a specified local surface size distribution $F_j$ (including the parameters $D_s$ and $\sigma_{sg}$ that can be calculated from its moments) and the local shear velocity $u_s$. The formulation for mixtures reduces to a formulation for uniform material in the limit as $\sigma_{sg} \rightarrow 0$. The mobility of the sediment mixture is controlled by an exponent $\beta$. The choice $\beta = 0$ corresponds to equal mobility (Parker et al., 1982), which would yield no downstream sorting. In the relation of Parker (1990a,b) $\beta$ is set equal to 0.0951, a value which renders coarser surface grains somewhat less mobile than finer grains, and is thus capable of driving downstream fining under aggradational conditions (see also Egiazaroff, 1965; Wiberg and Smith, 1989). The reader is referred to the original reference for details.

The bedload transport relation of Parker (1990a) is restricted to grain sizes that are too coarse to participate substantially in suspension. The lower bound of 2.0 mm suggested in the original paper is also employed here. It is seen from Fig. 1 that 66 percent of the feed material is coarser than this size. Sand is treated conceptually as throughput load; it fills the interstices of the gravel, any excess being transported farther downstream in suspension. This assumption appears reasonable for Runs 1 to 3. Paola and Seal’s (1995) consideration of the effect of patchiness is not included, in that the narrowness of the channel precluded their formation.

5 Discretization and solution technique

The numerical methods used in the present analysis are not new; space limitations preclude more than a brief description here. More details can be found in Cui et al. (1995), which was written as an appendix to this paper.

5.1 Fully coupled model for uniform material

In the case of uniform material (12b) drops out, and $q_T$ in (12a) simply denotes the volume bedload transport rate per unit width. The fully coupled method is then based on governing equations (1), (2) and (12a) which can be written in the following compact form:
\[ \frac{\partial \mathbf{w}}{\partial t} + \frac{\partial \mathbf{f}}{\partial x} + \mathbf{c} = 0 \]  

(17)

where

\[ \mathbf{w} = \begin{pmatrix} \eta \\ h \\ u \end{pmatrix}, \mathbf{f} = \begin{pmatrix} \frac{1}{1 - \lambda_p} q_r \\ uh \\ \frac{1}{2} u^2 + gh + g\eta \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ u^2 / h \end{pmatrix} \]  

(18a,b,c)

Equation (17) defines a system of hyperbolic equations which has three eigenvalues, all of which are real for typical problems of interest. They are solved using the MacCormack predictor-corrector method, using a form for numerical viscosity due to MacCormack and Baldwin (1975). The scheme is explicit and second-order accurate in both time and space.

5.2 Decoupled iterative scheme for uniform material and sediment mixtures

In the decoupled formulation (1) and (2) are excluded from the formulation for \( \mathbf{w} \). Equations (12a,b) are then written in the following compact form, into which a fourth order numerical viscosity has been introduced;

\[ \frac{\partial \mathbf{w}}{\partial t} + A \frac{\partial \mathbf{f}}{\partial x} + \mathbf{c} = -\mu \frac{\Delta x^4 \partial^4 \mathbf{w}}{\Delta t \partial x^4} \]  

(19)

Here

\[ \mathbf{w} = \begin{pmatrix} \eta \\ L_n F_j \end{pmatrix}, \mathbf{f} = \begin{pmatrix} \frac{1}{1 - \lambda_p} q_r \\ -f_{1,j} \end{pmatrix}, A = \begin{pmatrix} 1 & 0 \\ -f_{1,j} & 1 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 0 \\ (-f_{1,j}) \frac{\partial L_n}{\partial t} \end{pmatrix} \]  

(20a,b,c,d)

The formulation for uniform sediment is obtained simply by dropping (12b), so that the expressions in (20a–d) become scalar. Equation (19) is then discretized using a central difference scheme in space and a version of the Crank-Nicholson scheme in time. The resulting form requires an iterative solution, as described in Cui et al. (1995). Equations (1) and (2) for the flow are solved independently from (19), using time as a surrogate relaxation parameter to iterate to the steady state flow over the bed specified at each time step.

5.3 Boundary conditions

As mentioned above, the equations for uniform material have three eigenvalues, all real with two positive and one negative. The implication is that in implementing the fully coupled method, two boundary conditions are needed at the upstream end and one at the downstream end. In accordance with the three runs in SAFL, this is accomplished by setting the water discharge per unit width \( uh \) equal to the imposed value \( q_u \), the volume sediment feed rate per unit width \( q_r \) equal to the imposed
value \( q_T \) at the upstream end and the water surface elevation \( \xi = \eta + h \) equal to the imposed value at the downstream end \( \xi_w \). Note that the downstream boundary condition corresponds precisely to a weir at the downstream end of the flume, i.e. the actual configuration of the experiment. When other variables are required at the boundaries they are obtained from the method of characteristics. The boundary conditions for the decoupled scheme are essentially the same, except that they apply to different equations. In solving (1) and (2), \( q_w \) is imposed at the upstream end and \( \xi_w \) is imposed at the downstream end. In solving (12a,b), the values \( q_T \) and \( p_H \) corresponding to the sediment feed rate are imposed at the upstream end. Where other values are needed at the boundaries they are again evaluated from the method of characteristics.

### 6 Test of the decoupled numerical model against the SAFL runs

The decoupled numerical model was used to predict the evolution of SAFL Runs 1, 2 and 3. These experiments were introduced earlier; the configuration is shown in Fig. 2. Water elevation is controlled at the downstream end by an adjustable tailgate located 60 m downstream of the sediment feed point. The gravel front was not allowed to migrate more than 40 m downstream of the feed point in any of the runs. A modest amount of sand was typically carried across the front in suspension and deposited downstream. The tailgate, however, was not affected by sediment deposition, and served to maintain constant base level for the duration of each run.

The initial bed condition for each run was that of flow over a flat bed bare of sediment. The flow discharge was set with a standard orifice meter and held constant for the duration of the run. Sediment was then introduced at a constant rate at the upstream end, where it formed an asymmetric wedge. Most of the sediment moved downstream to form a mildly upward concave profile, but a small amount of sediment fell upstream at an angle close to the angle of repose, so allowing the wedge to build upward. The relatively small amount of sediment in the upstream portion of the wedge was heavily biased toward the coarsest grains in the sediment mix. As a result it was necessary to adjust the feed size distribution used in the numerical model to exclude the upstream wedge. This adjustment is described in detail in Toro-Escobar et al. (1996b). In addition the material finer than 2 mm was removed from the feed, in order to satisfy the constraint of the Parker (1990a,b) bedload transport model. The adjusted grain size distribution is shown in Fig. 1; it has a geometric mean size \( D_m \) of 12.8 mm, and a geometric standard deviation \( \sigma_m \) of 3.62.

<table>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>U</th>
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<td>49</td>
<td>49</td>
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<td>0.50</td>
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Toro-Escobar et al. (1996a) back-calculated the mean porosity $\lambda_p$ of the deposit of Run 3 from the data on feed rates and bed profiles and the equations of sediment continuity. The value of 0.47 obtained by them is an effective value which is larger than the actual value because the sand has been removed, as outlined above. This value was adopted for all three runs considered here. The water discharge per unit width $q_{sw}$, adjusted total gravel feed rate $q_{fr}$ (excluding sand) and tailgate water surface elevation $\xi_{sd}$ used as boundary conditions are all given in the Table; the definition of $\xi_{sd}$ is given in Fig. 2.

Results of the numerical model for each run are given in Figs. 3, 4, and 5. In each of these figures the predicted profiles of bed elevation $\eta(x, t)$, substrate geometric mean size $\bar{\psi}_{sub}$ and the substrate sizes $\psi_{sub, 90}$ and $\psi_{sub, 10}$ are plotted for the length of the deposit and at several times during the run. In addition, the predicted water surface profile $\xi(x, t)$ is plotted for the end of the run. Values of $\bar{\psi}_{sub}$, $\psi_{sub, 90}$ and $\psi_{sub, 10}$ were obtained from the model using the substrate fractions just below the surface layer $f_{sf}$ predicted from (13) and (14).

(a)

![Image](image_url)

(b)

![Image](image_url)

Fig. 3. Simulated and measured results for Run 1.
(a) water surface and bed elevation.
(b) characteristic grain size $\psi$ of the deposit.
Fig. 4. Simulated and measured results for Run 2.
   a) water surface and bed elevation.
   b) characteristic grain size $\psi$ of the deposit.

All these parameters are compared against data in the figures. In order to understand the plots it is necessary to understand how the data on substrate size distribution were obtained. The bed surface profiles shown in Fig. 3–5 were used to define standard time lines. At the end of the run, substrate samples were taken between each of the standard time lines. It follows that the size distribution so obtained represents the average of the substrate deposited between the two time lines in question. The predicted values of $\bar{\psi}_{\text{sub}_{0}}$, $\psi_{\text{sub}_{90}}$ and $\psi_{\text{sub}_{10}}$ are at these time lines whereas the measured data represent average values between the time lines. A necessary condition for perfect agreement, then, is that the measured points for these parameters fall in between the predictions for the two bounding time lines.

The agreement between measured and predicted bed and water surface elevation is seen to be excellent for Runs 2 and 3, and good for Run 1. The agreement as regards $\bar{\psi}_{\text{sub}}$ and $\psi_{\text{sub}_{10}}$ is also quite good for all three runs. The predicted values of $\psi_{\text{sub}_{90}}$ are typically 6 to 15 percent higher than the observed ones, a consistent but relatively modest error.
As pointed out earlier, the gravel transport model of Parker (1990a,b) has not been adjusted in any way to obtain this agreement. The definitions used here for $k_z$ and $L_a$ are slightly different from that of Parker (1991a,b), but not to the extent that the predictions of the model would be affected noticeably. The only added empiricism based on the experiments themselves is the use of (14) to characterize the material transferred to the substrate. This formulation is used due to the lack of any other formulation with an empirical or theoretical base. It was determined from Run 3, but gives reasonable agreement in Runs 1 and 2 as well.

The observed and predicted variation of flow Froude number $F$, in time and space are shown for Run 2 in Fig 6. Similar plots are obtained for Runs 1 and 3. Of interest is the fact that the observed flow is slightly supercritical almost everywhere upstream of the front, whereas the calculated flow is slightly subcritical everywhere upstream of the front. In light of the good agreement apparent in Fig. 4, the discrepancy would be of only passing significance, if it were not for the concern that a decoupled model may not perform accurately for a Froude number near unity. As is shown below, the
decoupled model can handle conditions resulting in mildly supercritical flow with no obvious numerical difficulty. The issue of coupled versus decoupled models is thus explored in the next section.

![Graph](image)

Fig. 6. Variation of Froude number $F_r$ in time and space for Run 2.

7 Coupled versus decoupled model

As mentioned earlier, several researchers have speculated that a decoupled formulation might fail if a) flow Froude numbers are close to unity or b) boundary values are rapidly varying in time. While only condition a) applies to the modeling of the SAFL narrow runs, it was decided to test the results of the coupled model versus the decoupled model for both conditions for the case of sediment with uniform size $D$. In applying the numerical model to uniform sediment, roughness height $k_s$ was set equal to 3.5 $D$. The results reported here are typical of those obtained in the course of a broader effort. The case of a Froude number near unity is considered first. The imposed conditions are intentionally set to be similar to those of the SAFL narrow runs, except that a) the sediment is uniform with grain size $D = 10$ mm and b) the sediment feed rate is adjusted so as to obtain Froude numbers at the upstream end very close to unity. For example, $q_w$ is set equal to 0.161 m$^3$/s, i.e. the value for Runs 1 $\sim$ 3 as shown in the Table. In this specific case, the calculated upstream Froude numbers are 1.086 and 0.974 respectively for the coupled and decoupled models. As can be seen in Fig. 7, the difference between the two models is negligible.

Cases with imposed time variations in upstream and downstream boundary conditions are considered next using the same value of $D$ as above. First, the input water discharge per unit width $q_w = q_w(t)$ is allowed to vary, with sediment feed rate $q_{sf}$ and downstream water surface elevation $\xi_d$ held constant. Hydrograph spikes were superimposed over a base water discharge $q_{w0} = 0.161$ m$^3$/s. As shown in Fig. 8a, the hydrograph includes spikes at hours 4, 6, 8 and 12, with the corresponding water discharge doubled within 2, 5, 10 minutes and 1.5 hours respectively. Fig. 8b demonstrates that the difference in the predictions between the coupled and decoupled model is negligible. Fig. 9 summarizes the boundary conditions for two additional cases. In one of these input water and sediment discharge are held constant at 0.161 m$^3$/s and 0.0002 m$^3$/s, respectively, and downstream water surface elevation $\xi_d$ is allowed to vary about a base value of 0.4 m, as described in the figure. In the other case input water discharge and downstream water surface elevation are held constant at 0.161 m$^3$/s and 0.4 m, respectively, and input sediment discharge is allowed to vary about a base value of 0.0002 m$^3$/s. In both cases the differences between the coupled and decoupled models were found to be minor.
Fig. 7. Comparison of the two models in case of upstream end Froude number close to unity.

Fig. 8. Comparison between the predictions of the fully coupled and decoupled models in a case for which the upstream hydrograph undergoes rapid variation. 
a) upstream hydrograph, b) water surface and bed profiles.
This excellent agreement between the decoupled and coupled models under conditions where the former might be expected to break down suggests that decoupled model is more versatile than might be expected from the literature. It also justifies its use as regards the SAFL runs. It does not demonstrate conclusively that the decoupled model never breaks down in the expected regions.

8 Test of the numerical model against a run with uniform sediment

The sediment transport model of Parker (1990a,b) can also be used in the limiting case of uniform sediment. In order to test it for this case, the first author performed an experiment in a small flume at SAFL. The flume was 35 cm deep, 15.5 cm wide and 10 m long. The sediment had a geometric mean size \( D \) of 2.07 mm and a geometric standard deviation of 1.23, a value small enough to justify classifying the sediment as uniform. The values of \( q_w \), \( q_T \) (= \( q_f \) in this case) and \( \xi \) are given under Run U in the Table. In this simulation the roughness height \( k_g \) is taken as equal to 4.5 \( D \).

The results of the coupled and decoupled numerical models are compared against the data in Fig. 10a. There is again negligible difference between the predictions of the coupled and decoupled models. The general shapes of the predicted profiles are reasonable. The models underpredict the location of the front for the time \( t = 10 \) min., but predict it accurately for the two later times. In addition, both the bed and water surface are predicted reasonably accurately over the downstream half of the profile upstream of the front. In the upstream half, however, both bed elevation and slope are noticeably underpredicted.

In order to understand the reason for this, several parameters in the Parker (1990a,b) bedload transport formulation were arbitrarily modified. Of particular interest is the reference Shields stress \( \tau'_{rgo} \), which corresponds loosely to a threshold Shields stress for the onset of significant bed motion based on the surface geometric mean size. This parameter takes the value 0.0386 in the formulation of Parker (1990a,b), based on field data for heterogeneous gravel. As shown in Fig. 10b, the agreement with data is markedly improved if this parameter is arbitrarily increased to 0.0550. In this case the predicted upstream Froude number is 1.27 for the decoupled model and 1.52 for the coupled model, as compared to an observed value of 1.41.
Fig. 10. Simulated and measured results for Run U, showing water surface and bed elevations.

a) \( \tau_{r,rgo} = 0.0386 \), as in the original formulation of Parker (1990a,b), b) \( \tau_{r,rgo} = 0.0550 \)

It is not argued here that the value of \( \tau_{r,rgo} \) in the formulation should be modified for uniform sediment, as one experiment is not sufficient to draw a general conclusion. It is rather concluded that the formulation must be used with caution when applied to uniform sediment, because in this particular case the value in the Parker (1991a,b) formulation is 30 percent lower than a value that gives excellent agreement with the data.

9 Conclusion

A decoupled numerical model of bed aggradation of heterogeneous sediment was developed to study downstream fining of gravel in rivers. It was tested against three large-scale experiments on downstream fining. The model uses the one-dimensional St. Venant equations, a Keulegan resistance relation, the gravel transport relation of Parker (1990a,b), and the formulation of the Exner equation for sediment continuity of mixtures given in Parker (1991a,b). An empirical formulation was used in the model to predict interfacial exchange fractions of material transferred to the sub-
strate as the bed aggrades (Toro-Escobar et al., 1996b). In the absence of any other information, this formulation was developed from one of the experiments against which the model was tested. Other than this, no attempt was made to fit the model to the data.

The agreement with the experimental data using heterogeneous sediment is good, suggesting that the model describes correctly the phenomena of aggradation with a prograding front and downstream fining. The model does not describe downstream fining due to the development of local patchiness, or local spatial variation in the grain size distribution due to bar structure. The width-depth ratios of the runs were well below the threshold value for the formation of bars, however.

A similar comparison against data pertaining to the aggradation of uniform sediment is not as good. The reason for this may be that the reference Shields stress in the Parker (1990a,b) relation is about 30 percent lower than a value that gives excellent agreement.

The observed and computed upstream Froude numbers for all the above experiments were close to unity. It has been suggested that a decoupled model might break down under such conditions. It has also been suggested that a decoupled model might fail when applied to the case of rapidly varying boundary conditions. With this in mind a fully coupled version of the numerical model was developed for uniform sediment, and compared against the decoupled model as applied to uniform material. The performance of the two models was virtually identical even for Froude numbers near unity, and even when the boundary conditions varied strongly in time. The results suggest that some of the concerns about the use of a decoupled model, while not necessarily invalid, may have been overstated.

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