RUNNING HEAD: CONTROLS ON SHORELINE MIGRATION

TITLE: EXPERIMENTAL MEASUREMENT OF THE RELATIVE IMPORTANCE OF CONTROLS ON SHORELINE MIGRATION

WONSUCK KIM 1, 2, 3, CHRIS PAOLA 1, 2, 3, VAUGHAN R. VOLLER 1, 2, 4, AND JOHN B. SWENSON 5

1National Center for Earth-surface Dynamics, University of Minnesota, Minneapolis, MN 55414, U.S.A.
2St. Anthony Falls Laboratory, University of Minnesota, Minneapolis, MN 55414, U.S.A.
3Department of Geology and Geophysics, University of Minnesota, Minneapolis, MN 55455, U.S.A.
4Department of Civil Engineering, University of Minnesota, Minneapolis, MN 55455, U.S.A.
5Department of Geological Sciences and Large Lakes Observatory, University of Minnesota, Duluth, MN 55812, U.S.A.

Corresponding Author: WONSUCK KIM (E-mail: kimx0826@umn.edu)

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ABSTRACT

Shoreline position in sedimentary rocks is a sensitive recorder of the interplay of several controlling factors. The most important of these are thought to be the “stratigraphic trinity”: eustatic sea level, subsidence, and sediment supply. In ancient rock sequences, it is generally difficult to disentangle the effects of these variables. Here we analyze the relative influence of sea level, subsidence, sediment supply, and other controlling variables on shoreline migration in an experimental basin equipped with a subsiding floor. The experiment used a linear-hinge type subsidence profile for which the rate was kept constant in time, constant overall sediment supply, and base-level variation on two time scales that were first applied separately and then superimposed. Although base level was the only controlling variable that was externally varied in time, the base-level changes induced changes in other variables indirectly (e.g., by changing the partitioning of sediment between the fluvial and offshore segments of the system).

We examine the relative importance of measured direct and indirect changes in all the governing variables through the use of a moving-boundary equation for shoreline migration. When measured values are used for all the variables in the equation, shoreline migration rate throughout the run can be predicted with a maximum $R^2$ of $> 0.92$. Starting with this optimal prediction of the observed shoreline behavior, we successively replace variables in the equation with their run-averaged values, degrading the prediction. The relative loss of prediction accuracy as each variable is replaced is a measure of the importance of that variable in accounting for the observed shoreline migration. By this measure, base level is the most important variable, followed in turn by sediment supply to the foreset, geometry of the foreset, and the average subsidence rate over the foreset. From the
shoreline migration equation, we also derive a quantitative version of the “$A/S$ ratio” often applied in sequence stratigraphy. The new formulation reduces to a form comparable to the traditional descriptive $A/S$ ratio if changes in foreset slope, gain or loss of sediment to the fluvial system, and spatial variation in subsidence rate are all negligible.

**INTRODUCTION**

The roles of controls on developing stratigraphic sequences have been debated since the early days of modern stratigraphy (e.g., Sloss 1962). Of these controls, eustatic sea level (Vail et al. 1977; Pitman 1978; Jervey 1988; Van Wagoner et al. 1988; Wilgus et al. 1988; Posamentier et al. 1988; Posamentier and Vail 1988), tectonic movement (Watts 1982; Summerhayes 1986), sediment supply (Galloway 1989a, 1989b; Thorne and Swift 1991; Schlager 1993), and basin physiography (Steckler et al. 1993; Helland-Hansen and Gjelberg 1994; Helland-Hansen and Martinsen 1996) have been proposed as the most important determinants of an evolving sedimentary basin within a time and space framework. However, questions have persisted about precisely how stratal patterns develop in response to these primary controls and which control, or combination of controls, exerts the dominant influence (e.g., Sloss 1962; Heller et al. 1993; Steckler et al. 1993). A major obstacle in answering these questions is the lack of high-resolution data that independently constrain the key governing variables (i.e., geo-history of tectonics, eustasy, and sediment discharge).

Here we use experimental stratigraphy (Paola et al. 2001) that models sedimentation associated with independent variations in sediment supply, absolute base-level change, and rates and geometries of subsidence to address these issues. The experimental stratigraphy was produced in the
Experimental EarthScape facility (XES) at St. Anthony Falls Laboratory of the University of Minnesota to investigate formation of stratigraphy under controlled conditions. The experimental study allows us to read the causes (input controls) and the effects (sedimentary record) precisely.

In the experiment we report on here, the only externally imposed time variable was base level, which is the experimental equivalent of eustatic sea level. Changes in base level produced transgressive, regressive, aggradational, and degradational stratigraphic units, which record migration in the shoreline position. As the shoreline migrates, key stratigraphic surfaces are delineated, allowing us to identify sequence boundaries. Thus, for base-level-controlled stratigraphy, sequence boundaries and stratal patterns can be described in the context of the shoreline trajectory (Helland-Hansen and Martinsen 1996).

Our objectives in this paper are (1) to investigate the shoreline trajectory as a key record in the experimental stratigraphy as a function of changing rate of accommodation (or rate of relative sea-level change: sum of rates in base-level change and subsidence), sediment supply, and basin topography, (2) to address the shoreline translation as a moving boundary problem by an extension of the mathematical model of Swenson et al. (2000), (3) to measure the input parameters of the mathematical model from the experimental data, and (4) to evaluate the relative importance of the controls in the mathematical model in generating the observed shoreline migration.

**EXPERIMENTAL DESIGN**

The XES basin develops strata under known variations in sediment discharge, rates and geometries of subsidence, and absolute base-level change. In this paper we describe the experiment
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(XES 02-1) designed to investigate the effect of slow, rapid, and superimposed base-level cycles on shoreline migration and stratigraphic response, under conditions of passive margin type subsidence. The XES 02-1 run used an area 5.72 m long and 2.98 m wide, one quarter of the entire basin (Fig. 1). The floor of the experimental basin comprises 108 independent subsidence cells that allow spatially variable subsidence. Subsidence is caused by gradual removal of a well-sorted, pea-size gravel from the bottom of each hexagonal cell. The gravel basement is covered by a flexible, thin rubber membrane so that input sediment and water are kept separate from the underlying gravel layer. Base level is controlled independently with a siphon attached to a movable constant-head tank in the downstream end of the basin. A more detailed description of the XES facility is available at [http://www.geo.umn.edu/orgs/seds/Sedi_Research.htm](http://www.geo.umn.edu/orgs/seds/Sedi_Research.htm) and has been reported in Paola (2000), Heller et al. (2001), Paola et al. (2001), Sheets et al. (2002), and Strong et al. (2005).

Sediment and water were mixed and fed from a single point source (Fig. 1). The sediment feed rate was 0.0182 m³/hr, the water discharge was 1.5 m³/hr, and their rates were kept constant throughout the experiment. The sediment mixture was composed of 63% quartz sand (110 µm), 27% coal sand (bimodal: 460 and 190 µm), and 10% kaolinite. The coal has a specific gravity of 1.3, whereas quartz has a specific gravity of 2.65, so the coal grains are substantially more mobile than the quartz grains and serve as a proxy for fine-grained clastics. The mixture of quartz and coal was similar to that used previous experiments (e.g., Sheets et al. 2002), but kaolinite was added to improve the stability of the deposit.

Subsidence rates increased linearly downstream in the basin so as to produce simple linear-hinge type subsidence (Fig. 1). The rates of subsidence were held constant in time. The maximum subsidence rate in the downstream end of the basin was 3.7 mm/hr.

Absolute base-level change (i.e., absolute change of water surface level in the “ocean” part of...
the basin) of the XES 02-1 experiment included sinusoidal base-level cycles with two time scales (Fig. 2A). The first slow base-level cycle lasted 108 hours, beginning at runtime 26 hr. It was followed by a rapid base-level cycle lasting 18 hours beginning at runtime 144 hr. Base level was stable before the first slow cycle and between these two cycles (gray area in Fig. 2A); this stable base level is the experimental datum. The second part of the experiment comprised six rapid cycles superimposed on one slow cycle, beginning at runtime 202 hr. The slow component had a duration of 108 hours, and the six rapid base-level cycles were each 18 hours in duration. Absolute base level reached a minimum level of 0.11 m below initial at runtimes 80 and 153 hr on the low-frequency and high-frequency sinusoidal curves, respectively, and a minimum of 0.21 m below initial level at runtimes 247 and 265 hr during the third and fourth superimposed cycles (Fig. 2A). The maximum rate of the base-level change was 22.3 mm/hr on the falling and rising inflection points of the second superimposed cycle and fifth superimposed cycle, respectively.

**INITIAL EXPERIMENTAL RESULTS**

Subsidence and base-level controls were initiated (at runtime 0 hr) after allowing the shoreline to migrate out to the midpoint of the basin (Fig. 2B, average shoreline position: 2.77 m downstream at the beginning of the run) without subsidence and base-level fluctuation (Fig. 2C, bottom thick deposit is from this pre-run).

Fluvial and submarine topography was collected 101 times during the 310-hour duration of the experiment (Fig. 2B). A laser altimeter and an ultrasonic sonar transducer were used to record the elevations of the fluvial surface (0.1 mm vertical resolution) and the submarine surface (1 mm
vertical resolution), respectively. The grid spacing of the surface elevation is 0.05 m in the dip ($x$) direction and 0.01 m in the strike ($y$) direction (Fig. 1B). The subsidence was monitored every four minutes by an array of 108 pucks, one for each subsidence cell, buried in the gravel just below the rubber membrane. These pucks were connected through a fluid multiplexer to a sensitive pressure transducer that measured change in the hydrostatic pressure in the water lines, and thus the puck elevations with 0.1 mm vertical resolution. Eight collected data sets were excluded because they were either repeats of the topographic scans due to a relative long pause of the run or uncompleted scans. Thus, 93 out of 101 collected data sets are presented in this paper.

The surface elevation of each scan was laterally averaged to produce a single surface profile parallel to the dip direction (Fig. 2C). We reconstructed a time series of stratigraphic evolution composed of these depositional surface profiles from the 93 topographic scans. At each time step, the prior surface profile was subsided and the new profile added, deleting eroded points as needed. The shoreline (i.e., as a point on the 2-D cross section) was determined as the intersection of absolute base level and the latest surface in each scan time (Fig. 2C). As the shoreline migrated (e.g., regression, transgression, aggradation, and degradation), its trajectory was traced as a cross-sectional path (Helland-Hansen and Martinsen 1996). The shoreline trajectory as well as the surface profiles progressively moved downward as the basement subsided. Figure 2C is a final reconstructed stratigraphy with the shoreline trajectory at the end of the run (runtime 310 hr). A time series of the reconstructed stratigraphy including all 93 time frames is available as a movie file through the website at http://www.geo.umn.edu/orgs/seds/Movies/avg_strata.mov. We note that the reconstructed stratigraphic time lines do not account for internal deformation such as faults. The actual deposit shows that faulting, though present, did not significantly affect the accommodation
Over the 310-hour duration of the experiment, the shoreline shifted 1.58 m landward from where it started (Fig. 2B). This overall pattern of the shoreline translation can be accounted for the effects of capture ratio (Paola et al. 1992) and autoretreat (Muto and Steel 1997; Swenson et al. 2000). The capture ratio is a ratio of the rate of creation of space by tectonic movement to sediment influx; roughly if the ratio is greater than one, the basin is underfilled; if the ratio is less than one, the basin is overfilled (Paola et al. 1992). The capture ratio of the XES 02-1 was set at slightly higher than 0.5 so the experimental basin was underfilled, causing overall landward migration of the shoreline. In particular, the finite slope of the delta front, which is usually higher than the fluvial slope, though lower than the submerged angle of repose, is maintained by deposition over the delta front. The increasing volume on the delta front to be filled as the front elongates means that an increasing fraction of the overall sediment supply to the basin is deposited offshore, causing a slow transgression termed “autoretreat” (Muto and Steel 1997; Swenson et al. 2000). Figure 2B illustrates the autoretreat trajectory of the XES 02-1 run calculated using the Swenson et al. (2000) model assuming constant base level and steady subsidence as described above.

There were significant overshoots of the shoreline in the transgression of the individual base-level cycles (Fig. 2B), i.e., the shoreline had shifted landward more than the autoretreat trajectory when the absolute base level came back to its initial level at the end of base-level cycle. For instance, the overshoot is 0.89 m relative to the predicted autoretreat trajectory at the end of the slow base-level cycle out of the total 1.62 m shoreline excursion between the maximum regression and transgression in the slow cycle. It becomes increasingly inaccurate to use the autoretreat trajectory of the entire run for the last base-level cycles as a reference for the overshoot calculation because the nonlinear response of shoreline to the base-level cycles moves the mean shoreline position.
successively farther from the autoretreat curve. However, the effect of autoretreat is negligible for the rapid cycles. Overshoot at the maximum transgression is 1.19 m out of the total 1.68 m excursion in the rapid base-level cycle and 1.79 m out of the total 2.82 m excursion in the superimposed base-level cycle.

The shoreline trajectory on a cross section is formed by a series of discrete segments that connect two consecutive shoreline positions (Fig. 2C). Each segment of the shoreline trajectory can be represented as a vector, in which the vertical component of the vector is the sum of rates of subsidence, \( \sigma(s) \), and base-level change at the shoreline, \( \sigma(s) + \dot{Z}_w \), while the horizontal component is the shoreline migration rate, \( \dot{s} \) (Fig. 3). The vertical component is the rate of change in relative sea level at the shoreline so the vector provides a relationship between changes in the relative sea level and shift of the shoreline (i.e., regression or transgression). We refer to the vector sum of these two components as the \textit{incremental stratigraphic vector}. Figure 4 illustrates changes in the magnitude of the vertical and horizontal components of the incremental stratigraphic vector during the XES 02-1 run, in which regression corresponds to the first and fourth quadrants (i.e., right half of the graph), transgression to the second and third quadrants (i.e., left half of the graph), and pure aggradation to the vertical axis. The plot (Fig. 4) overall indicates negative correlation of vertical aggradation of the sediment at the shoreline to horizontal shift of the shoreline, as noted by Schlager (1993).

Figures 5B, 5C, 5D, and 5E represent individual paths (see the definition sketch in Fig. 5A) that connect the ends of the incremental stratigraphic vectors for the slow base-level cycle, rapid cycle, first three cycles of the superimposed cycle, and last three cycles of the superimposed cycle, respectively. Distance from the center of the graph to the path denotes the speed of the shoreline migration (i.e., length of the incremental stratigraphic vector). The different distances of the paths
from the center of the graph in Figures 5B and 5C indicate different speeds of the shoreline migration during the slow and rapid base-level cycles. The elongated path of the rapid cycle denotes high shoreline speed.

Normal regression (both the vertical and horizontal components are positive) is dominant only in the slow base-level cycle while forced regression (the vertical component is negative but the horizontal component is positive) is dominant in the last of the base-level cycles (Fig. 5). The path of the slow cycle in Figure 5B plots on the first quadrant, denoting relative sea-level rise and thus normal regression, but the paths of the rapid cycle and superimposed cycles (Fig. 5C, 5D, and 5E) plot on the fourth quadrant, denoting relative sea-level fall and thus forced regression. A type 2 unconformity developed at the falling inflection point in the slow base-level cycle while type 1 unconformities developed toward the end of base-level cycle in the last of the rapid and superimposed base-level cycles (Fig. 2C).

The incremental stratigraphic vector rotates clockwise or anticlockwise in a base-level cycle (Fig. 5A); clockwise rotation generates concave-up shoreline trajectories for transgression (the shoreline becomes increasingly aggradational and less transgressive with time) and convex-up for regression (the shoreline becomes less aggradational and either increasingly regressive for normal regression with time or decreasingly regressive for forced regression with time). Anticlockwise rotation generates convex-up shoreline trajectories for transgression and concave-up trajectories for regression. The regression trajectories in all base-level cycles are convex up because the vector rotates clockwise (Fig. 5B, C, D, and E). The paths of transgression in Figures 5B, 5C, and 5E rotate clockwise so the shoreline trajectory is concave up, whereas the path of transgression in Figure 5D rotates anticlockwise so the shoreline trajectory is convex up.

For a given magnitude (length) of the incremental stratigraphic vector, the angular spread of the
rotation path is a measure of the curvature: narrow rotation paths denote low curvature of the shoreline trajectory (e.g., regression in the right-hand side of Fig. 5C); wide rotation paths denote high curvature of the shoreline trajectory (e.g., regression in the right-hand side of Fig. 5D).

Curvature of the trajectories of the rapid (Fig. 5C) and first three superimposed cycles (Fig. 5D) indicates different rates of erosional processes on the fluvial surface during the forced regression. The narrow rotation of the transgression path in the rapid cycle represents low curvature (Fig. 5C); the wide rotation of the transgression path in the last three superimposed cycles represents high curvature (Fig. 5E). The curvature of the transgressive shoreline trajectory corresponds to the rate of sediment accumulation on the fluvial surface; the last three superimposed cycles have higher rates of sediment accumulation than does the rapid cycle.

During the first three superimposed cycles, the low preservation potential of fluvial deposits is easily recognized because of both high erosion on the fluvial surface, which is related to high curvature of the shoreline trajectory during the forced regression, and the low rate of the sediment accumulation related to the convex-up shoreline transgression (see both Fig. 2C and Fig. 5D). However, the last three superimposed cycles preserved more fluvial sediments because of the high increase of accumulation rate in transgression related to the high curvature of the concave-up trajectory (see both Fig. 2C and Fig. 5E). The scale of the path of the incremental stratigraphic vector indicates the speed of regression and transgression, and the direction and shape of the rotation are related to the preservation potential of the fluvial sedimentary record.

**MATHEMATICAL MODEL**
Our shoreline model is a slight extension of the work of Swenson et al. (2000). The model begins with the mass balance between sediment influx and deposition within a basin in two dimensions. We derive a mass-balance equation using a simple geometric argument. It turns out that each term of the equation can be related to one of the primary controls of stratigraphy listed above.

Consider a cross section of a sedimentary basin such as that illustrated in Figure 6, in which the subsidence rate, sediment discharge, and topographic elevation are strike averaged, i.e., integrated normal to the direction of sediment transport. Conservation of global sediment mass can be expressed as

$$\int_{-\infty}^{\infty} \int_{0}^{\eta} s_{so} \, dx \, dt + \int_{-\infty}^{\infty} \int_{0}^{\eta} s_{ts} \, dx \, dt = \int_{-\infty}^{\infty} \int_{0}^{\eta} s_{ss} \, dx \, dt ,$$

(1)

where \( s_{so} \) is the sediment influx delivered to the basin at \( x = 0 \), \( \eta \) is topographic elevation of the sediment surface, \( b \) is elevation of the subsiding basement, and \( s \) is downstream position of the shoreline. The sedimentation is divided into fluvial and submarine deposits by the shoreline, which is a moving boundary. We use the Exner equation of sediment conservation

$$\frac{\partial (\eta - b)}{\partial t} = -\frac{\partial q_{s}}{\partial x}$$

(2)

to derive a mass-balance equation for sediment passed across the shoreline

$$q_{ss}(s,t) = \int_{s}^{\infty} \frac{\partial}{\partial t} (\eta - b) \, dx = \int_{s}^{\infty} \frac{\partial}{\partial t} (\eta - b) \, dx + \int_{s}^{\infty} \frac{\partial}{\partial t} (\eta - b) \, dx ,$$

(3)

where \( q_{ss}(s,t) \) is the volume rate of sediment transport per unit width at the shoreline and \( u \) is the position of the delta toe. In the context of shoreline dynamics, definition of the toe requires some care. We define the toe position as the boundary between proximal and distal submarine sedimentation on the foreset and bottomset, respectively, which is typically associated with a change
in depositional slope. Only the foreset deposit participates directly in controlling shoreline migration by removing sediment via avalanches.

Defining the toe position is also critical to the application of $A/S$ ratio concept (Sloss 1962; Curray 1964; Jervey 1988; Posamentier et al. 1988; Galloway 1989a, 1989b; Schlager 1993; Muto and Steel 2000), i.e., the ratio of rate of the creation of accommodation ($A$) or relative sea level to the rate of sediment supply ($S$). This conventional definition used as a predictor of the shoreline translation (Sloss 1962; Curray 1964; Jervey 1988; Posamentier et al. 1988; Galloway 1989a, 1989b; Schlager 1993); basinward shift of deltaic shoreline occurs when $A/S < 1$, landward shift of deltaic shoreline when $A/S > 1$, and stationary deltaic shoreline when $A/S = 1$. However, autoretreat (Muto and Steel 1997, 2000) occurs under the condition of $A/S = 1$ in terms of the conventional definition. Muto and Steel (1997) debated the dimensional error and definition of the conventional $A/S$ ratio concept. We analyze this in more detail in the discussion section.

For the foreset deposit as defined above, the sediment discharge that controls shoreline migration, $q_f$, is distributed as

$$q_f(s,t) = \int_s^t \frac{\partial}{\partial t} (\eta - b) dx.$$  (4)

We assume a linear topography of the delta front with slope, $S_f$, that can vary in time. The geometry of the foreset is

$$\eta = Z_m - S_f (x - s),$$  (5)

where $Z_m$ is the base level. Equations 4 and 5 can be combined using the relations

$$-\frac{\partial b}{\partial t} = \sigma(x) \quad \text{and} \quad \frac{\partial (\eta - b)}{\partial t} \bigg|_{a} = 0,$$  (6)
where $\sigma$ is a spatially variable rate of tectonic movement measured positive downward, to yield a balance between $q_{sf}$ and sediment distribution under given conditions of the rate of base-level change, tectonic subsidence, and change in the delta-front morphology

$$q_{sf}(s, t) = (u - s) \left( \frac{dZ_{bl}}{dt} + S \frac{ds}{dt} - \frac{1}{2} (u - s) \frac{dS_f}{dt} \right) + \int_s^u \sigma(x) dx . \quad (7)$$

Equation 7 can be written as an integro-differential equation for horizontal shoreline migration:

$$\frac{d}{dt}(u - s)S_f = q_{sf}(s, t) - (u - s) \frac{dZ_{bl}}{dt} - \int_s^u \sigma(x) dx + \frac{1}{2} (u - s)^2 \frac{dS_f}{dt}, \quad (8)$$

where $ds/dt$ is the shoreline migration rate; if $ds/dt > 0$, the shoreline shifts basinward; if $ds/dt < 0$, the shoreline shifts landward. $ds/dt$ is equal to the horizontal component of the incremental stratigraphic vector defined previously. Equation 8 indicates that regression and transgression are determined by the four different terms in the right-hand side (RHS), each representing a geological process. RHS of Equation 8 denotes how much sediment (first term) deposits in the space created by base-level change (second term) and subsidence (third term), and how the shape of the clinoform evolves during sedimentation (fourth term). Equation 8 is consistent dimensionally: each term has units of areal change per unit time.

**APPLICATION TO EXPERIMENTAL DATA**

Our next step is to use experimental data from the XES 02-1 run to predict shoreline migration via Equation 8. Of the parameters in Equation 8, the base-level change $dZ_{bl}/dt$, and subsidence $\sigma(x)$, are imposed, but the sediment discharge $q_{sf}$, foreset horizontal length $(u - s)$, and foreset
slope $S_f$, are dependent variables, i.e., they are determined by the dynamics of the experimental system and must be measured from the experimental data. Determination of the shoreline and toe positions was necessary to measure $q_f$ and $S_f$. We defined the shoreline as the intersection of base level and the sediment surface. Defining the delta toe is more difficult. The goal is to identify the part of the offshore deposit that directly influences shoreline migration. The offshore limit of the delta toe is not clear, even in the simple experimental stratigraphy, because the clinoform deposit thins out gradually from the shoreline and/or the offshore deposit can be discontinuous because of sediment bypassing on the steep preexisting bottom topography (Fig. 2C).

For many observed clinoform geometries, both in the experiment and in the field, there is no clear slope break in slope at the foreset base and thus the toe position, $u$, is ambiguous. We determined $u$ heuristically relative to the preexisting sediment surface by finding the minimum foreset slope, $S_f$, satisfying the condition

$$\frac{d}{dt} \int (q - b)dx = \frac{d}{dt} \int [Z(w - S_f(x - s)) - b]dx .$$

This condition represents the actual delta-front geometry with a linear surface of slope $S_f$ such that an equivalent sediment volume is deposited across the time interval. This criterion works for most clinoform geometries (Fig. 7A) but does not work for those where offshore sediments bypass the steep slope of the preexisting shelf edge (Fig. 7B). For these cases, we assumed a synthetic flat bottom (dotted line in Fig. 7B) at the rollover of the closest shelf edge from the shoreline. The toe position was then determined as the intersection of the linear surface with the given flat bottom where Equation 9 is satisfied.

The discharge for the sediment passing through the shoreline and depositing on the foreset area
\( q_f \), and the slope of the delta front \( S_f \), were measured based on the delta-toe position (Fig. 7). This discharge, \( q_f \), was constrained between the shoreline and toe positions by Equation 4 and linearly averaged between two consecutive scan times. Input variables for calculating Equation 8 are summarized in Figure 8. The ratio of the sediment discharge at the shoreline \( q_s \), to the constant feed rate of sediment \( q_o \), at the upstream end indicates how much sediment is delivered to the shoreline from the sediment point source (Fig. 8A). The ratio \( q_s / q_o \) generally increases with the rate of fall of base level. The maximum value of \( q_s / q_o \) occurred at the falling inflection points of each base-level cycle when the fluvial deposit was strongly incised and bypassed. The fraction of the foreset sedimentation in the total offshore sedimentation \( q_f / q_s \), denotes the contribution of the offshore deposit to the shoreline migration (Fig. 8A). This contribution tends to diminish with falling base level because the delta front migrates close to the steep slope on the preexisting shelf edge, causing more submarine sediment transport to the deep floor. The overall pattern of toe migration with time followed that of the shoreline, but the horizontal distance between the shoreline and toe generally increases with the distance from the shelf edge and the amount of sediment delivered to the shoreline (Fig. 8B). Within a given base-level cycle, the delta-toe position generally migrated out to the last shoreline maximum in the previous base-level cycle (Fig. 8B) because the maximum downstream position of the shoreline in the previous base-level cycle became the rollover of the underlying shelf edge for the next base-level cycle. In general, during a base-level cycle, there is a 45 degree phase shift between the base level and foreset slope change such that the foreset slope reaches a maximum at the falling inflection points (Fig. 8C).
MODELING RESULTS

Using Equation 8, we calculated the horizontal component of the shoreline migration using values for the terms in the RHS estimated as described above. The input data for the terms were measured and averaged under the assumption that their change is linear between two consecutive scan times. Comparison of the modeled horizontal shoreline migration with observations is shown in Figure 9H. The extended 2-D model accounts for over 90% of the observed variability.

Next we evaluate the relative importance of the terms in Equation 8 in controlling shoreline migration. We order the various simplifications of Equation 8 into eight levels, Level 0 being the simplest and Level 7 being the complete solution shown in Figure 9H. The hierarchy of the eight different levels is ranked by improvement in error (Fig. 9). The error was calculated by

\[
\text{error} = \frac{\sum_{1}^{N} \left( \frac{ds}{dt} \right)_{\text{obs}} - \left( \frac{ds}{dt} \right)_{\text{calc}}}{\sum_{1}^{N} \left( \frac{ds}{dt} \right)_{\text{obs}}} \tag{10}
\]

where \( \left( \frac{ds}{dt} \right)_{\text{obs}} \) is the observed shoreline migration rate, \( \left( \frac{ds}{dt} \right)_{\text{calc}} \) is the calculated shoreline migration rate, and \( N \) is the number of measurements. Initially, we set all input variables of the model constant at their average value over all 93 scans. We then successively replaced variables one by one with their measured, time-dependent values and traced maximum improvements among the possible choices.

We begin at Level 0 with eustatic variation because it gives the least error of any single variable. The condition for Level 0 can be written as
\[
\frac{ds}{dt} = \frac{q_u}{(u-s)S_f} - \frac{1}{S_f} \frac{dZ_{bl}}{dt} - \frac{\sigma(s)}{S_f},
\]
(Level 0)

where the overbar denotes a time average of measured data. Level 0 also assumes no gain or loss of sediments from the fluvial system. The Level 0 result shows that eustatic change alone accounts for about 40% of the observed shoreline behavior (Fig. 9A). If the tectonic subsidence is negligible, Level 0 indicates that base-level fall \((dZ_{bl} / dt < 0)\) causes the shoreline regression in any cases but base-level rise \((dZ_{bl} / dt > 0)\) is compensated by the sediment supply over the foreset area, thus only large enough supply (i.e., \(q_u / (u-s) > dZ_{bl} / dt\)) causes the shoreline regression. There is an overall shift of the data cloud in the plot to the right-hand side because the sediment supply at the upstream end, \(q_u\), is higher than that involved in shoreline migration. The systematic shift and error diminish when an averaged foreset sediment supply, \(\bar{q}_{sf}\), is applied rather than \(q_u\) in Level 1 (Fig. 9B). The simplification of Level 1 is equal to that of Level 0 except we use the averaged foreset sediment supply, \(\bar{q}_{sf}\):

\[
\frac{ds}{dt} = \frac{\bar{q}_{sf}}{(u-s)S_f} - \frac{1}{S_f} \frac{dZ_{bl}}{dt} - \frac{\sigma(s)}{S_f}. \quad \text{(Level 1)}
\]

Level 2 captures the effect of allowing the foreset slope to vary in time. The condition is represented as

\[
\frac{ds}{dt} = \frac{\bar{q}_{sf}}{(u-s)S_f} - \frac{1}{S_f} \frac{dZ_{bl}}{dt} - \frac{\sigma(s)}{S_f}. \quad \text{(Level 2)}
\]

The result of Level 2 illustrates a remarkable reduction in systematic error, in which slope of the best fit curve improves to 1.03 from its Level 1 value of 1.50 (Fig. 9C). The rate of the transgression varies for the same rate of base-level rise according to the foreset slope. Thus, a decrease in the
foreset slope during transgression increases the rate of shoreline migration landward. This increasing rate occurs for regression in the same way with increasing foreset slope.

Level 3 denotes addition of time variation in the sediment supply (Fig. 9D). The condition has measured changes in time of the foreset sediment supply, \( q_{sf} \), foreset slope, \( S_f \), and base level, \( Z_h \):

\[
\frac{ds}{dt} = \frac{q_{sf}}{(u-z)S_f} - \frac{1}{S_f} \frac{dZ_h}{dt} - \frac{\sigma(s)}{S_f}.
\]  

(Level 3)

There is noteworthy improvement in predicted regression (plots on the right hand side of the graph) between Levels 2 and 3, which reflects the influence of the sediment supply on the regression process. The result of Level 3 suggests that accurately accounting for eustatic variation, foreset slope change, and sediment supply over the foreset depositional area accounts for over 80% of the observed shoreline migration pattern in the experimental stratigraphy.

Levels 4 and 5 successively add time variations of the subsidence rate at the shoreline and toe-shoreline distance, respectively (Fig. 9E and F). The tectonic subsidence plus the base level controls relative sea level, thus fast subsidence contributes the shoreline transgression. The subsidence rate in Level 4 is determined at the shoreline and assumed to be spatially constant over the foreset but varying with time:

\[
\frac{ds}{dt} = \frac{q_{sf}}{(u-z)S_f} - \frac{1}{S_f} \frac{dZ_h}{dt} - \frac{\sigma(s)}{S_f}.
\]  

(Level 4)

The result barely changes between Levels 3 and 4 because the magnitude of spatial variation of the hinge type subsidence rate in this run does not vary much within a cycle.

Level 5 adds variable foreset length:

\[
\frac{ds}{dt} = \frac{q_{sf}}{(u-z)S_f} - \frac{1}{S_f} \frac{dZ_h}{dt} - \frac{\sigma(s)}{S_f}.
\]  

(Level 5)
Relative to Level 4, there is visible improvement in the slow shoreline migration both landward and basinward, which is plotted near the center of the graph (Fig. 9F).

Level 6 adds the explicit time derivative of foreset slope, $dS_f / dt$, (as opposed to parametric change in $S_f$ with time, which was included in Level 2) (Fig. 9G):

$$\frac{ds}{dt} = \frac{q_d}{(u-s)S_f} - \frac{1}{S_f} \frac{dZ_{id}}{dt} - \frac{\sigma(s)}{S_f} + \frac{(u-s)}{2S_f} \frac{dS_f}{dt}.$$  

(Level 6)

The overall error in fast to slow shoreline migration rate is corrected when the additional term is applied.

Spatial variation of the hinge type subsidence is left for the last improvement to get the final Level. At Level 7, all of the time variations available are used to compute the shoreline migration rate correctly (Fig. 9H):

$$\frac{ds}{dt} = \frac{q_d(s,t)}{(u-s)S_f} - \frac{1}{S_f} \frac{dZ_{id}}{dt} - \frac{1}{(u-s)S_f} \int_s^u \sigma(\xi) d\xi + \frac{(u-s)}{2S_f} \frac{dS_f}{dt}.$$  

(Level 7)

**DISCUSSION**

*Relative Importance of Stratigraphic Variables*

The extended version of the mathematical condition of Swenson et al. (2000), Equation 8, predicts the shoreline migration of the XES 02-1 run with an error of 0.233 (Level 7 in Fig. 9H). We successively accounted for time-dependent variables to evaluate the order of relative importance of stratigraphic variables. Among the possible substitutions of average values with time variation, we selected one that gives a maximum improvement, and then successively chose substitutions that best
improved the result, keeping the previous substitutions (Fig. 10). The steps gradually corrected the modeling results up to Level 7, which accounts for all variables. The first step has only eustatic variation, which makes the best improvement among the possible choices. This reflects not only the original design of the XES 02-1 run, in which base level was the only imposed variable, but also the critical effect of base-level change on stratigraphy. The sediment supply to the shoreline and clinoform geometry follow as the most important variables to decrease error after the eustatic variation. Interestingly, the foreset slope varied strongly with time and significantly influenced shoreline response. The foreset slope generally became steeper during the regression and shallower during the transgression, which contributed to increase of magnitude of the shoreline migration rate. The sediment supply also plays a major role in the $A/S$ ratio concept, which we discuss in the next section. The subsidence rate at the shoreline and the horizontal toe-shoreline distance are ranked as the next most important variables. However, variation of the foreset length throughout the run and variation of the subsidence rate in the foreset area were relatively too small to generate large responses in shoreline shift. We can explain this lack of sensitivity by noting that the spatial variation of the subsidence rate was approximately 0 to the maximum of 3.7 mm/hr, whereas the rate of base-level change varied with time between 0 to 22.3 mm/hr. Finally, the term added to account for the spatially variable subsidence rates over the foreset gave the smallest reduction in error.

Possible Sources of Error

The modeling result shown in Figure 9H has a residual error. Possible sources of error from major to minor can be summarized as follows. (1) The topographic scan devices have limited time and space resolution and cannot measure the entire area of the experimental tank. (2) We assumed that all variables change linearly between two consecutive scan times. (3) We did not account for
three-dimensional effects such as incised valleys, faults, channel migration, and lobe switching because the model uses laterally averaged topographic data.

**A/S Ratio**

Balance or imbalance of the rate of changes in accommodation (A) and the rate of sediment input (S), as embodied in the “A/S ratio”, has long been considered a primary factor in the development of stratigraphic architecture (e.g., Sloss 1962; Curray 1964; Posamentier and Vail 1988; Posamentier et al. 1988; Van Wagoner et al. 1988; Kendall and Lerche 1988; Jervey 1988; Galloway 1989a, 1989b; Thorne and Swift 1991; Schlager 1993; Steckler et al. 1993). The original idea can be traced back to Sloss’s (1962) explanation of a relationship among “Q (the quantity of detritus supplied to the depositional site per unit time)”, “R (the receptor value that the available volume of the depositional site below base level created per unit time by subsidence)”, and “D (dispersal that the rate at which transporting agents tend to remove material from the depositional site)”. Sloss suggested that the stratal geometry of continental margins is explained as a function of these geological processes. \( Q > R \) indicates that the supplied sediment is sufficient to fill up the space, the excess of the sediment bypasses the depositional area, and regression occurs (i.e., \( A/S < 1 \)). In this case, R and \( (Q - D) \) are governing factors of the shape of sedimentary record. If \( Q = R \), then indicates that all the supplied sediment is consumed to fill up the depositional site (i.e., \( A/S = 1 \)), and \( Q \) and R are governing factors. Finally, \( Q < R \) indicates the basin accommodates all the supplied sediment at the depositional site and transgression takes place (i.e., \( A/S > 1 \)). In this case, \( Q \) and R again are governing factors.

The ideas of “depositional site” and “D (dispersal factor)” are worth noticing in Sloss’s explanation. As he reported, a certain limited area as the depositional site should be defined to
compare \( Q \) and \( R \) with dimensional consistency. Sediment transported outside of this depositional site should be extracted from \( Q \), thus \((Q - D)\) becomes one of primary controls on the developing stratal pattern inside the depositional site. The mathematical model in this paper is consistent with Sloss’s idea in terms of both defining the depositional site as a specific subset of the depositional system and subtracting dispersal (i.e., bypassed) sediment from the total sediment input to a site. The depositional site is consistent with the foreset area and \( D \) is consistent with \( q_u - q_f \), but in our model the foreset area, \( u - s \), and sediment discharge over the foreset, \( q_f \), are defined by the two clearly defined moving boundaries, the shoreline, \( s \), and the toe, \( u \).

One problem with the \( A/S \) ratio as conventionally defined is that it is not clear precisely what \( A \) and \( S \) mean or that the ratio is dimensionless (Muto and Steel 1997, 2000). On the basis of Equation 8 we offer a quantitative and physically consistent \( A/S \) ratio:

\[
\frac{(u-s)S_f}{q_f(s,t)} \frac{ds}{dt} = 1 - \left[ (u-s)\frac{dZ_{uf}}{dt} + \int_s^u \sigma(x) dx - \frac{1}{2} (u-s)^2 \frac{dS_f}{dt} \right] \frac{q_f(s,t)}{q_f(s,t)},
\]

where the second term on the RHS is a form of \( A/S \) ratio. This definition is dimensionally consistent and physically meaningful to evaluate the regression and transgression processes: The second term on the RHS of Equation 11 less than one indicates regression \((ds/dt > 0)\); greater than one indicates transgression \((ds/dt < 0)\); equal to one indicates a stationary shoreline position \((ds/dt = 0)\).

If the rate of foreset slope change is negligible, the new definition of the \( A/S \) ratio (i.e., term two of the RHS of Equation 11) can be written as
If loss or gain of sediment by the fluvial system and spatial subsidence variability are negligible, then Equation 12 can be simplified further

\[
\frac{(u-s) \frac{dZ_{bl}}{dt} + \int_{s}^{u} \sigma(x) dx}{q_{so}(s,t)}
\]  

Equation 13 is applicable where the marine depositional site \((u-s)\) is fairly well defined and stationary, spatial variability of the subsidence is low, and fluvial deposition or erosion is unimportant within the time scale analyzed.

**CONCLUSIONS**

Judging from analysis of XES 02-1 experimental stratigraphy, base level is the most important control on shoreline regression and transgression, followed in turn by sediment discharge to the foreset, geometry and horizontal length of the foreset, and the average subsidence rate over the foreset. It seems clear that we could answer quantitatively which geological process or set of processes exerted the major control on the overall shoreline migration during the run.

We note that the relative importance determined in this paper may not generalize to systems
forced differently than in this experiment. In particular, in the experiment the only imposed time variable was base level; all changes in the other variables (e.g., shoreline sediment supply, foreset slope, subsidence following over the depositional site) were secondary results of the base-level change. However, this data set and method provide an example of how to evaluate the hierarchy of geological processes and how precise interpretation is possible in the hierarchy. We have found that rigorous definition of the depositional site on the foreset area is of critical importance not only in comparing scales of the stratigraphic controls but also in defining the $A/S$ ratio. Our analysis leads to a new, precise form of the $A/S$ ratio, presented in Equations 11 to 13, which is dimensionally consistent and should allow more accurate stratigraphic prediction.

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FIGURE CAPTIONS

Fig. 1. A) Schematic experimental setup and B) plan view of the XES basin showing the area where topographic data are collected.

Fig. 2. A) Absolute base-level change over time throughout the XES 02-1 run. B) Change in the shoreline downstream position from the topographic scans during the experiment. C) Reconstructed stratigraphy with the shoreline trajectory at the end of the run.

Fig. 3. Schematic definition of the incremental stratigraphic vector. Shoreline migration rate is the horizontal component, and the rate of relative sea-level change at the shoreline position is the vertical component.

Fig. 4. Changes in the horizontal and vertical components of the incremental stratigraphic vector during the XES 02-1 run.

Fig. 5. A) Definition sketch for the incremental stratigraphic vectors and the vector path in a base-level cycle. Cross plot of the incremental stratigraphic vectors and their path in B) slow base-level cycle (note: vertical and horizontal axes are different scale than in other graphs), C) rapid base-level cycle, D) first three superimposed base-level cycles, and E) last three superimposed base-level cycles. Sketches in right-top and left-bottom corners of parts B, C, D, and E indicate evolution of sediment surface for each given condition. The arrow in these sketches is the trajectory of the shoreline migration, and the dotted line is the eroded surface.
Fig. 6. Two-dimensional idealized basin defining the parameters used in the mathematical model.

Fig. 7. Definition sketch for method developed for mapping the effective toe position, \( u \), and foreset slope, \( S_f \). A) Representative example of application of the method and B) exception to the application of the method, which requires a synthetic flat bottom (dotted line) to define the toe position.

Fig. 8. Input variables for the mathematical model. A) Ratio of sediment discharge at the shoreline, \( q_s \), to sediment feed rate from the single input source, \( q_{so} \), and ratio of foreset sediment discharge, \( q_{sf} \), to sediment discharge at the shoreline, \( q_s \). B, C) Records of, respectively, shoreline, \( s \), and toe positions, \( u \), and foreset slope, \( S_f \), and absolute base level, \( Z_{bl} \), during the experiment.

Fig. 9. Eight series of comparisons of the modeling results with the observed shoreline migration rate using successive levels of available data showing the relative importance of stratigraphic variables. The data added at each level are explained in the text and in Figure 10.

Fig. 10. Eight steps of improvement in error of the modeling results showing the relative importance of each stratigraphic variation applied to correct the result.
(A) Base level

(B) Shoreline position

(C) Reconstructed stratigraphy
vertical component: $[\sigma(s) + \dot{Z}_u]$

incremental stratigraphic vector

horizontal component: $[\delta]$

subidence $\sigma(x)$

base level $\dot{Z}_u$

time line after subsidence at time $t$

time line before subsidence at time $t - \Delta t$

new time line after subsidence at time $t$
(A) Incremental stratigraphic vectors and vector path

(B) Slow cycle

(C) Rapid cycle

(D) First 3 superimposed cycles

(E) Last 3 superimposed cycles
\[ \eta(x, t) \]

\[ b(x, t) \]

\[ Z_b(t) \]

\[ q_{so} \] sediment supply

\[ s(t) \] shoreline

\[ S_f(t) \] foreset slope

\[ \sigma(x) \] subsidence

\[ b(x, t) \] basement elevation

\[ u(t) \] delta toe

\[ u(t) \] shoreline
(A) Run time [hr] vs Ratio [0] for different ratios of $q_{ss}/q_{so}$ and $q_{sf}/q_{ss}$. (B) Distance [m] vs Slope [0], Depth [-m] showing shoreline, foreset slope, and toe elevation. (C) Slope [0], Depth [-m] plot illustrating the relationship between slope and depth changes.
(A) Level 0
\[ y = 0.9742x, \quad R^2 = 0.4247, \quad \text{Error} = 0.790 \]

(B) Level 1
\[ y = 1.5005x, \quad R^2 = 0.6828, \quad \text{Error} = 0.578 \]

(C) Level 2
\[ y = 1.0276x, \quad R^2 = 0.7933, \quad \text{Error} = 0.452 \]

(D) Level 3
\[ y = 1.0645x, \quad R^2 = 0.8872, \quad \text{Error} = 0.347 \]
(E) Level 4

\( \frac{dy}{dt} \text{ observed [m/hr]} \)

\( \frac{dy}{dt} \text{ calculated [m/hr]} \)

\( y = 1.0588x, R^2 = 0.8885, \text{Error} = 0.338 \)

--

(F) Level 5

\( \frac{dy}{dt} \text{ observed [m/hr]} \)

\( \frac{dy}{dt} \text{ calculated [m/hr]} \)

\( y = 0.9784x, R^2 = 0.8784, \text{Error} = 0.321 \)

--

(G) Level 6

\( \frac{dy}{dt} \text{ observed [m/hr]} \)

\( \frac{dy}{dt} \text{ calculated [m/hr]} \)

\( y = 0.9427x, R^2 = 0.9201, \text{Error} = 0.245 \)

--

(H) Level 7

\( \frac{dy}{dt} \text{ observed [m/hr]} \)

\( \frac{dy}{dt} \text{ calculated [m/hr]} \)

\( y = 0.9566x, R^2 = 0.9277, \text{Error} = 0.233 \)
\[ \frac{dZ_b}{dt} \Rightarrow \frac{dZ_s}{dt} \]

\[ q_{bs} \Rightarrow q_{ds} \]

\[ S_i \Rightarrow S_i S_f \Rightarrow q_s \Rightarrow q_f \]

\[ \overline{\sigma(s)} \Rightarrow \sigma(s) \]

\[ (u-s) \Rightarrow (u-s) \]

additional term

\[ (u-s)\sigma(s) \]

\[ \int_s^u \sigma(x)dx \]