THE INFLUENCE OF TRANSPORT FLUCTUATIONS ON SPATIALLY AVERAGED TOPOGRAPHY ON A SANDY, BRAIDED FLUVIAL FAN

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ABSTRACT: Determination of the transport ("diffusion") coefficient, the main parameter of most forward models for generating fluvial stratigraphy, requires finding the average slope required to transport the total sediment load delivered to a given point for a given water discharge. Finding this value, in turn, requires averaging the substantial fine-scale local variability in transport capacity that one encounters in most natural rivers. The problem is especially acute for braided rivers, in which the local capacity varies strongly in time and space as channels migrate, flow shifts from one part of the channel network to another, and confluences, which account for a disproportionate share of sediment flux, form and dissipate. Here, we present a model for computing spatially averaged sediment flux in a sandy braided river system. Coupled with sediment mass balance, the sediment-flux model leads to the usual diffusion equation for surface topography. The problem of indeterminacy of channel width is dealt with by using an empirical constant value of 1.8 for the mean nondimensional (Shields) stress. We test the model by applying it to a mine-tailings fan in which all independent parameters (sediment flux, water flux, grain size, deposition pattern) are well known and constant. The statistical parameters needed to determine the transport coefficient are determined from independent measurements of the river network on the fan. Using these inputs, the model predicts the fan topography well. The model suggests that, for a highly active braided system such as this one, the effect of the fluctuations in sediment flux can increase total sediment flux by a factor of two to four relative to what would be predicted from mean values alone. The data also suggest, however, that some of the key statistical parameters vary significantly downstream along the fan. This variation may result from downstream variation in grain-size distribution, sediment flux, or both.

INTRODUCTION

For some years, various forms of the diffusion equation have been used to represent the large-scale, spatially averaged transport and deposition of sediment by river systems (Begin, 1987; Flemings and Jordan, 1989; Jordan and Flemings, 1991; Paola et al., 1992). (Note that the connection between the fluvial processes represented and diffusion itself is purely formal; fluvial transport is not mechanistically "diffusional"). In many cases, the diffusion equation is simply assumed, but derivation of it from first principles (Paola et al., 1992) is useful both in clarifying the assumptions that must be imposed to obtain diffusion and in understanding the structure of the primary parameter of the equation (the "diffusion coefficient," which we will refer to here as the transport coefficient). As applied to sedimentary basins, the equation reads:

\[
\sigma + \frac{\partial \eta}{\partial t} = \frac{-1}{C_0} \nabla H \left[ \nabla \cdot H (\eta) \right]
\]

where \(\eta\) is the elevation of the sediment surface relative to some fixed, arbitrary datum, \(t\) is time, \(\sigma\) is subsidence rate, \(C_0\) is volumetric sediment concentration in the bed, \(\nabla H\) is the horizontal divergence operator, and \(\nu\) is the transport coefficient.

The main parameter that sets the transport coefficient is the time-averaged water supply to the system. In essence, the diffusion equation for topography boils down to the Exner equation for mass balance:

\[
\sigma + \frac{\partial \eta}{\partial t} = \frac{-1}{C_0} \nabla H (\tilde{q}_w)
\]

plus a relation among topographic slope \(S\), unit water discharge \(q_w\), and unit sediment discharge \(q_s\):

\[
S = \nu_0 \frac{q_s}{q_w}
\]

where \(\nu_0\) is a dimensionless coefficient such that \(\nu = \nu_0 q_w\).

It is evident from the derivation of the equation that a good deal of physics is wrapped into the single transport coefficient \(\nu_0\), which determines the slope required to transport a given flux of sediment with a given flux of water. One could think of it as a measure of the efficiency of the river system. We will refer to \(\nu_0\) as the intrinsic transport coefficient.

The purpose of this paper is to determine what sets the intrinsic transport coefficient for a well-constrained sandy braided river system. The heart of the problem is that equation 2 is a relation among three time-averaged quantities, but in detail the transport processes in most rivers are variable in time and space. This is particularly true of braided rivers, which show wide ranges of local properties (e.g., velocity, depth, sediment flux) even if input fluxes are steady. The constant reconfiguration of the flow pattern gives the system an apparently stochastic nature, even though in a local, short-term sense it is completely deterministic. The variable nature of the flow field cannot simply be ignored: several key quantities, such as flow resistance and sediment flux, are known to depend nonlinearly on other variables; however, collecting and averaging tens or hundreds of measurements of sediment flux each time one has to estimate average sediment flux is not practical. Here, we develop a spatially averaged model for flow and sediment transport in a sandy
braided river, building on the approach of Paola (1996). The overall strategy is to try to identify key statistical quantities, determine how they control spatially averaged properties of system, and begin work on how these quantities are set by the dynamics of the stream system.

In a river of fixed, known width and uniform cross section (i.e., a flume), established techniques exist for determining the slope for given water and sediment fluxes (and hence $v_0$). In contrast, in the general case of freely braiding rivers, the flow is both spatially unconstrained and constantly reconfiguring itself. The width is not only not known, it is not even clear how it should be defined. The velocity and other basic quantities vary in space and time in an apparently stochastic manner. The problem, then, has two interrelated parts: definition of spatially averaged mean-field equations, and provision of some constraint that allows specification of the total flow width.

FIELD AREA: TAILINGS-BASIN FAN OF THE ROLLING STONE MINE

Basin-scale stratigraphic models typically cannot be tested rigorously due to the difficulty of constraining both the temporal evolution of the stratigraphy and the boundary conditions. The analysis we developed was part of a study of a mine-tailings fan in northern Minnesota. The fan and the nature of the engineering problems associated with it are discussed in detail in Parker et al. (in press a, b). A map of the fan is shown in Figure 1. The fan system is an ideal natural laboratory for the study of basin filling processes. The mining company, referred to hereafter as Rolling Stone, supplies water and sediment at near-constant rates at the upstream end of the fan via two flumes (“launders”) at the apex of the fan. The water supply is much larger than any addition of water on the surface due to rainfall. The sediment supply ranges in size from fine gravel through clay (size), and consists almost entirely of crushed chert. The sediment is forced to deposit in a fixed area by raising dikes around the basin perimeter, so on average the rate of deposition in the system is constant in space and time. Just downstream of the feed flumes, the fan surface develops into a highly active braided stream system. This stream style is maintained over the length of the system until near the downstream end, where deposition of the finest fraction of the sediment supply occurs in ponds. The topographic and sediment-supply data for this study were obtained in 1992 for the subarea West Area 1 in Figure 1. In effect, the Rolling Stone fan acts as a small sedimentary basin with a spatially and temporally constant subsidence rate matched to a constant rate of sediment supply. The extremely active, fluctuating braided stream system on the fan produces a remarkably stable, convex surface topography when averaged over time (Figure 2). The problem before us is to model this surface topography given the known, constant fluxes of sediment and water.

SPATIALLY AVERAGED WATER AND SEDIMENT FLUXES: THEORY

The approach we use here is that proposed in Paola (1996), based on using turbulence as an analog for braiding. In both types of systems, internal fluctuations lead to localized events that dominate transport: the burst-sweep cycle dominating momentum transport in turbulence, and confluences dominating sediment transport in braided rivers. A Reynolds-like decomposition of the governing St. Venant, Exner, and sediment-flux equations leads to equations relating spatially averaged quantities of interest (in this case, slope) to averaged input values, with the addition of new terms arising from the averaging.

Here, we focus on modifying the theory for the Rolling Stone sandy alluvial fan. A basin filling model in radial coordinates adapted for fans such as this one is presented in Parker et al. (in press a). This paper presents models in two forms, one for

![Fig. 1.—Map of the Rolling Stone fan. Contours are in meters. The study area for this paper is West Area 1. Brick pattern is preexisting topography and the shaded lines are dikes.](image)

![Fig. 2.—Time sequence of downstream bed profiles for the study area with elevation given relative to an arbitrary datum. The high degree of self-similarity of these bed profiles indicates that the fan topography has evolved to a steady state adjusted to give steady, uniform deposition. Brick pattern represents preexisting topography.](image)
unchannelized flow and one for channelized flow. As applied to the tailings fan in Parker et al. (in press b), the channelized version links a diffusional segment describing the dynamics of the sandy part of the fan to a constant-slope segment that accounts for deposition of fines at the distal end of the deposit. The sand-dominated majority of the system is the zone of primary interest here. In Parker et al. (in press b), the final value of the intrinsic transport coefficient (\(v_0\) in equation 2) was determined by fitting the calculated form of the surface topography profile to the observed topography. Here we attempt to constrain it using a physical model of stochastic effects on sediment and water transport similar to that developed in Paola (1996), together with independent measurements of the necessary statistical properties of the Rolling Stone system.

We begin with a local relation for sediment flux. In Paola (1996) the transport relation used is the formula of Meyer-Peter and Müller (1948). This is reasonable for bed load–dominated gravel systems, but would be a poor choice here, where most of the sediment is sand and silt and the transport is dominated by suspension. A straightforward and well tested relation for sand that includes suspension transport is that due to Engelund and Hansen (1967), which can be written as

\[
\frac{q_s}{(s-1)gD^{1/2}} = \frac{0.05}{c_f} \left( \frac{\tau_0}{\rho(s-1)gD} \right)^{5/2}
\]

(4)

where \(q_s\) is the local volumetric sediment flux per unit width of active channel, \(s\) is sediment specific gravity, \(g\) is gravitational acceleration, \(D\) is grain size, and \(c_f\) is drag coefficient = \(\tau_0/\rho u^2\) where \(\tau_0\) is local boundary shear stress, \(\rho\) is fluid density, and \(u\) is local mean flow speed. The theory in Paola (1996) was developed for constant grain size, but in the case of the Rolling Stone fan and many systems of sedimentological interest, the grain size varies considerably over a section. If the shear stress and grain size were statistically independent, their probability distributions could be evaluated separately and their effects simply superimposed. Unfortunately, this is not a very realistic approximation; even casual observation suggests that it is common for high-stress regions of stream beds to be floored with relatively coarse sediment (Figure 3). The sampling program at the Rolling Stone fan allowed us to collect the data needed to evaluate this effect. We rewrite equation 4 as

\[
q_s = q_{s0} D_{50s}^{-1} \tau_0^{5/2}
\]

(5)

where \(D_{50s}\) is nondimensional grain size equal to \(D_{50}/D_{20}\), the angle brackets denote averaging over the entire cross section, \(\tau_s = \tau_0/\tau_0\), and \(q_{s0}\) is a reference transport rate for the cross section, given by

\[
q_{s0} = \frac{0.05}{c_f} \left( g(s-1) \right)^{-1/2} \left( \frac{\tau_0}{D_{50s}^{5/2}} \right)
\]

The transport rate averaged over the cross section is obtained from equation 5 as

\[
[q_s] = q_{s0} I_{\tau s}
\]

(6)

where \(I_{\tau s}\) is an integral function given by

\[
I_{\tau s} = \int \int D_{50s}^{-1/2} \tau_0^{5/2} f_{\delta s}(D_{50s}, \tau_s) D_{50s} d\tau_s
\]

in which \(f_{\delta s}\) is the joint probability density function (PDF) of dimensionless grain size and shear stress.

The sampling program for the tailings-basin project included evaluation of the local relationship between shear stress and grain size, which allows us to construct an approximate joint probability-density function for grain size and shear stress. This can then be used to evaluate equation 6. Anticipating that the lognormal distribution is a reasonable model form for the grain size, we introduce the transformation \(\delta_s = \log_2(D_{50s})\). The log-transformed joint probability density \(f_{\delta s}\) can be estimated from field data if it is decomposed as

\[
f_{\delta s} = f_{\delta s}(\delta_s, \tau_s) \times f_{\tau s}(\tau_s)
\]

(7)

where \(f_{\delta s}(\delta_s, \tau_s)\) is the conditional probability density function for \(\delta_s\) given a particular value of \(\tau_s\), and \(f_{\tau s}(\tau_s)\) is the PDF for \(\tau_s\). Paola and Seal (1995) showed that the gamma distribution provides a reasonable model form for the stress PDF; we will use it here as well.

Figure 4 shows field data relating local shear stress and median grain size; the correlation between stress and grain size is clear, as is the substantial variability of grain size for a given shear stress. The data suggest that the correlation between \(\delta_s\) and \(\tau_s\) may be accounted for by a relation of the form \(\delta_s = F(\tau_s) + \varepsilon\) where \(F\) is a least-squares fitted function and \(\varepsilon\) is a normally distributed random variable with zero mean. This allows the conditional probability density to be written as

\[
f_{\delta s} = \frac{1}{\sigma_{\delta s} \sqrt{2\pi}} e^{-\left[\delta_s - F(\tau_s)\right]^2 / 2\sigma_{\delta s}^2}
\]

(8)

The least-squares equation \(\delta_s = F(\tau_s)\) fitted to the data is shown on Figure 4. The variance \(\sigma_{\delta s}^2\) can be obtained from the correlation coefficient \(r^2\) as (Keeping, 1962)

\[
\sigma_{\delta s}^2 = \sigma_\delta^2(1-r^2)
\]

(9)

where \(\sigma_\delta^2\) is the variance of \(\delta_s\), which can be determined from field data.
Combining equations 6–9 with the gamma PDF for \( \tau_s \), gives

\[
I_{D \tau} = \int_0^\infty \int_0^\infty \left( \frac{2^{-\delta_\tau} \tau_s^{5/2}}{\sigma_\tau \sqrt{2\pi(1-r^2)}} \right) \left( \frac{\alpha^\alpha \Gamma^{-1}(\alpha) e^{-\alpha \tau_s^2}}{\Gamma(\alpha)} \right) d\delta_\tau d\tau_s
\]

(10)

where \( \Gamma \) is the standard gamma function and \( \alpha \) is a nondimensional measure of the inverse width of the stress distribution, given by \( \langle \tau_0 \rangle/\sigma_\tau^2 \) where \( \sigma_\tau \) is the standard deviation of the stress distribution. The joint probability density function \( f_{D \tau} \) in equation 10 is shown in Figure 5. Equation 10 is fully specified by the input parameters \( \sigma_\tau, r, \alpha \), and the empirically determined function \( F(\tau_s) \).

It is not practical to collect enough individual stress measurements to specify the stress PDF directly. Instead, we estimate the statistics of the stress distribution by using the square of the velocity as a proxy for stress. This assumes that the stress locally satisfies a standard drag law

\[
\tau_0 = \rho c_j u^2
\]

(11)

with \( c_j \) taken to be constant. As shown in Parker et al. (in press a), allowing \( c_j \) to vary in the Manning-Strickler form as \( c_j = c_{0j}(H/D)^{\rho} \) leads to a modified version of equation 3:

\[
S = \left( \nu_0^3 Q_s/Q_w \right)^{1/(1+\rho)}
\]

(12)

where \( Q_s \) and \( Q_w \) are total discharges of sediment and water, respectively. The nonlinearity in equation 12 leads to a weak nonlinearity in the final diffusion equation. As discussed in Parker et al. (in press b), measured local stress and velocity values from the Rolling Stone tailings basin give a best-fit \( p \) value of 0.12, but can be fit nearly as well with \( p = 0 \), eliminating the nonlinearity and allowing use of the squared velocity as a direct proxy for stress. As was done in Parker et al. (in press b), we will use \( p = 0 \) here.

The stress distribution can be further constrained if the mean stress can be estimated from physical reasoning; indeed, a physical constraint on the mean shear stress seems the most promising path in general for providing the additional relation required to determine flow width. In Parker et al. (in press b), values of the nondimensional Shields stress \( \theta = \tau_0/[\rho(s-1)gH] \) collected from the Rolling Stone fan were combined with values from single-channel and braided sand rivery reported in the data compendium of Church and Rood (1983). Although there is considerable scatter in the data, the overall trend is a remarkable constancy in \( \theta \) over a range of more than two orders of magnitude in slope. This suggests a constant-stress closure, \( (\theta) = \text{constant} \). The overall mean value of \( (\theta) \) determined in Parker et al. (in press b) was 1.8, and that value is used here as well.

The remainder of the theoretical development required for treating the Rolling Stone fan is very similar to that in Paola (1996). In spatially averaged form, the total water discharge \( Q_w \) is expressed as

\[
Q_w = W(\langle u \rangle) = W(\langle u \rangle) + W(\langle u' \rangle)
\]

(13)

where \( W \) is total wetted width of the cross section, \( h \) is local flow depth, and the primes denote local deviations away from the section-averaged value; e.g., \( h' = h - \langle h \rangle \). The quantity \( \langle u' \rangle \), which is analogous to a Reynolds stress in turbulence, represents a net transport of water by fluctuations in depth and velocity.
away from the local average. This value is expected to be positive because in general areas of higher depth also have higher velocity. We assume for the sake of simplicity that this transport term can be modeled as \( \langle u \theta' \rangle = \gamma(a)'(h) \), where \( \gamma \) is a parameter to be determined from field data.

The spatially averaged velocity and shear stress are related by

\[
\langle u \rangle = I_t \sqrt{\frac{\langle \tau_0 \rangle}{c_f}}
\]  

(14)

where \( I_t \) is another integral function given by

\[
I_t = \frac{\alpha}{T(\alpha)} \int_{-\alpha}^{\alpha} \int_{-\sqrt{2}}^{\sqrt{2}} e^{-a^2} dY
\]

This equation defines a spatially averaged effective drag coefficient for the system, given by \( c_f I_t \).

The final relation needed to close the problem is momentum balance for the flow. In Paola (1996) this is analyzed using the same Reynolds-decomposition technique used to obtain equation 13. The analysis is somewhat lengthy and will not be repeated here, but the final result is to recover the usual long-wavelength (normal flow) approximation to the momentum balance:

\[
\langle \tau_0 \rangle = \rho g \langle h \rangle \langle S \rangle
\]

(15)

Finally, combining the constant-stress closure with equations 6, 10, 11, 13, 14, and 15 gives us a relation for the intrinsic transport coefficient:

\[
v_0 = \frac{0.05(\theta)}{(s-1)(1+\gamma)e^{\beta(\alpha)} I_{DC} } I_t
\]

(16)

**FIELD DETERMINATION OF INPUT PARAMETERS**

The parameters required to specify equation 16 completely are \( \theta \), \( \gamma \), \( \beta \), and the inputs to the two integral functions \( I_t \) and \( I_{DC} \); \( \sigma_b, \gamma, \alpha \), and the empirically determined function \( F(\gamma_0) \). Values for \( r \) and \( F(\gamma_0) \) were previously discussed and are shown on Figure 4. To set \( \gamma \) one needs data on the correlation between local depth and local velocity. As discussed above, data on local velocity are also required to estimate the coefficient of variation of the shear stress, which determines \( \alpha \). Finally, data on the section-averaged grain-size distribution, together with analyses of local size distribution, are required to determine the standard deviation of local median sizes \( \sigma_b \).

Accordingly, we took samples at the Rolling Stone fan aimed at measuring these parameters. We established three sampling traverses across the fan system at distances of 1140, 1610, and 1940 m from the fan apex. The transects were taken on the east side of West Area 1 (Figure 1). We measured water depth and velocity at 1 m intervals across the width of each traverse, for a total of 678 data points. We also collected bulk sediment samples over each section for later sieve analysis. The main results of the traverses are given in Table 1. One point to note in Table 1 is that for each traverse the sum of the measured unit discharges \( q_u \) exceeds the actual total discharge \( Q_u \) of 7.5 m³/s. Although some of this could be due to measurement error, the main effect is the migration of the channels during the traverses. The braided system on the fan surface is extremely active, and migrated laterally as the traverses were being measured. In principle, this activity could act either to decrease or increase the measured discharge; during the measurement period the channels tended to migrate in the same direction as the traverses were being run, leading to oversampling of the entire flow system. There is no reason to think the oversampling was biased toward any particular range of flow conditions, so we do not believe the dimensionless statistics of the flow field (e.g., \( \alpha, \gamma \)) were affected by the oversampling.

When one surveys cross sections in a single-thread or confined braided river, the lateral boundaries of the survey are clear. This is not the case in an unconstrained braided river. One must decide how much of the flow field actually contributes to active sediment transport and system dynamics. One clearly does not want to include areas of standing water in overall transport statistics; however, as shown in Figure 6, there is no cutoff that defines a "dominant" channel in the Rolling Stone system. The total water (and by implication sediment) transport per unit width declines smoothly from the most active areas to zones of standing water.

**TABLE 1. SUMMARY OF MEASURED AND CALCULATED PARAMETERS FOR THE ROLLING STONE FLUVIAL FAN**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Upstream Section</th>
<th>Middle Section</th>
<th>Downstream Section</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstream distance (m)</td>
<td>1140</td>
<td>1610</td>
<td>1940</td>
<td>N/A</td>
</tr>
<tr>
<td>( DS0 ) (m)</td>
<td>0.242</td>
<td>0.103</td>
<td>0.081</td>
<td>0.045**</td>
</tr>
<tr>
<td>( Q_w ) (m³/s)</td>
<td>9.86</td>
<td>8.22</td>
<td>11.86</td>
<td>9.98</td>
</tr>
<tr>
<td>Total data points</td>
<td>187</td>
<td>259</td>
<td>252</td>
<td>N/A</td>
</tr>
<tr>
<td>( h ) (m)</td>
<td>0.070</td>
<td>0.0675</td>
<td>0.079</td>
<td>0.072</td>
</tr>
<tr>
<td>( u ) (m/s)</td>
<td>0.695</td>
<td>0.499</td>
<td>0.669</td>
<td>0.621</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.849</td>
<td>1.121</td>
<td>1.675</td>
<td>1.215</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.58</td>
<td>0.18</td>
<td>0.15</td>
<td>0.24</td>
</tr>
<tr>
<td>( I_{DC} )</td>
<td>4.97</td>
<td>2.14</td>
<td>1.69</td>
<td>2.93</td>
</tr>
<tr>
<td>( I_t )</td>
<td>0.869</td>
<td>0.897</td>
<td>0.929</td>
<td>0.898</td>
</tr>
</tbody>
</table>

*aData in the shaded cells are based on statistics for \( P = 0.99 \).

**This is the median size of the sediment pool, not the average of the three sections.*

**FIG. 6—Cumulative discharge vs. cumulative width, both as fractions of the total for the cross section, for the middle Rolling Stone cross-section.**
Our approach to this problem is to define significant statistical quantities, patterned loosely after the usage in gravity-wave statistics, where the "significant wave height" is defined as the average height of the highest one-third of all waves. We define a significant statistical quantity as the value of that statistic computed over all measurements points the sum of whose unit discharges accounts for some fraction P of the total discharge. Hence, for example, with :math:`P = 0.9` and a total of :math:`N` measurements, the significant variance of velocity would be obtained by (1) ranking all the depth-velocity pairs :math:`v_i, h_i` according to the value of :math:`q_{ai} = u_i h_i`, (2) starting with the largest :math:`q_{ai}`, finding the value of :math:`n` for which
\[ \frac{\sum_{i=1}^{n} q_{ai}}{\sum_{i=1}^{N} q_{ai}} = 0.9 \]
and (3) computing the variance of :math:`u` for the data set :math:`u_{1-n}`.

The choice of :math:`P` has a modest effect on the computed statistics; in general, variances decrease and mean values increase as :math:`P` is reduced. In our view it would be difficult to defend a choice of :math:`P` much less than 0.90 (about 60% of the total wetted width) or much beyond 0.99 (about 84% of total wetted width), but there is no obvious criterion for choosing a particular value within this range. We have calculated statistics for both of these :math:`P` values; only those for :math:`P = 0.99` are given in Table 1, but fan topography was estimated using both limits.

The values of the input parameters for equation 16 are given for each of the three traverses in Table 1, and Figure 7 shows a representative joint PDF of :math:`u` and :math:`h` for one traverse. The parameter values used for calculation of the fan topography were the averages of the values for the three traverses.

With the input parameters listed in Table 1, equation 16 is completely constrained. The only free parameter is the significance criterion :math:`P` used to define the range of values used to compute statistics, and, within the range :math:`0.9 < P < 0.99` that we consider physically reasonable, this exerts only a modest effect on the final values of :math:`v_0`. Once :math:`v_0` is computed, the sediment and water discharges and the map-view geometry of the deposit area completely specify the slope and surface topography via equations 1–3. In the present case, :math:`\partial \eta/\partial t` is constant in space because the fan topography is at steady state. [A complete development of the steady-state form of equation 1 in radial coordinates is presented in Parker et al. (in press b).]

The major remaining problem is to correct the total supplied load for washload material. It is customary to define washload as material that is present in the flow in much higher abundances than in the bed. Apart from being vague, this does not seem to us to get at what is really important physically about washload. Rather, we propose that the washload be defined as that fraction of the total supplied load that does not play a significant role in determining the bed slope. This means that if the washload were removed, the slope would not change. We are not aware of any detailed theory that would predict this, but it seems to us that two conditions could be used to partition the total load into a washload component and an "effective" or slope-determining component: (1) the Rouse number :math:`w_s/\kappa u_e` (where :math:`w_s` is the settling velocity, :math:`\kappa` is the Von Karman constant, and :math:`u_e` the friction velocity) for the washload must be much less 1 and (2) the actual flux of washload must be much smaller than the transport capacity (i.e., the flux of material of those sizes that the flow could transport given unlimited supply and no other material in the flow). Both conditions imply that for a depositional transport system the washload cutoff would vary down the system.

In the absence of a detailed downstream sediment budget for the Rolling Stone fan, we make a crude overall approximation based on the supplied size distribution (Figure 8) and typical midfan transport conditions (:math:`D_90 = 100 \mu m`, :math:`\tau_e = 1.8`). The cutoff for washload of :math:`D = 40 \mu m` used in Parker et al. (in press b) gives a median size for the washload of about 9 \mu m. The maximum value of the Rouse number for this size range is 0.018, and the actual flux is only 4% of capacity based on the median size and 19% based on the maximum size. Using the more traditional cutoff value of 60 \mu m, the median size of the washload becomes 11 \mu m. The maximum Rouse number is still only 0.08, and the actual flux is 6% of capacity based on median and 34% based on the maximum grain sizes. These numbers are still fairly conservative, and so a cutoff value of 60 \mu m is adopted here; however, we stress that this is still a fairly arbitrary choice, and that correct calculation of washload, preferably as a function of position, is an important topic that has received far less attention in the literature than it deserves.
As was done in Parker et al. (in press b), we assume that once all the sand has been depleted, the deposit is dominated by the fines. We model that section with a constant empirically determined slope of 0.0023. Slopes are computed for the sand-dominated zone using equations 1–3 and 16 until the computed slope becomes less than 0.0023; then the slope is set to that value. The model slopes and topography computed by integrating the slope are shown in Figure 9. Topography and slope are shown for statistics based on $P$ values of 0.9 and 0.99. We consider the agreement with observed topography to be good considering that no tuning was done to find the solution.

**DISCUSSION**

Transport coefficients in stratigraphic models are typically treated as free parameters. The results presented here suggest that this coefficient can be constrained, even for a relatively complex, dynamic river system. The problem, especially in ancient systems, is estimating the statistical parameters that measure the effect of fluctuations on section-averaged transport rates. The greatest obstacle to further progress in this area is lack of field data. Apart from the results presented here, we know of only one other data set from which transport statistics can be estimated for a braided river, that of Mosley (1982) for the Ohau River in New Zealand; nonetheless, the analysis presented here suggests that the effects of internal fluctuations on net sediment transport are strong. The mean ratio of the intrinsic transport coefficient $v_0$ calculated using the complete stochastic model to that computed without any of the statistical corrections (i.e., using equation 16 with $\gamma = 0$ and $I_t = I_{Ds} = 1$) is 2.25 based on $P = 0.9$ statistics, and 2.64 based on $P = 0.99$ statistics; the maximum values of the ratio obtained for the upstream cross section are 3.66 and 4.16, respectively. [Note this ratio is the coefficient $\alpha_{in}$ in Parker et al. (in press a, b).]

The computed statistics for the three sections measured here are comparable to those reported by Mosley (1982) for the Ohau, which is a much larger (mean annual flood discharge $Q_m = 280$ m$^3$/s) and coarser grained (bulk $D_{50} = 20$ mm) river than that of the Rolling Stone fan. Depth-velocity data supplied to the senior author by P. Mosley for a steady discharge $Q_m = 240$ m$^3$/s give $\alpha = 1.14$ and $\gamma = 0.3$, which are close to the values we found for the middle Rolling Stone traverse; however, the decrease in stress variance moving downstream in the Rolling Stone system, as indicated by the increase in $\alpha$ (Table 1), is noteworthy. This change suggests that the system is becoming less variable, and presumably less dynamic, going downstream. Two physical effects could account for this. The decreased stress variance could be related to decreasing variance in grain size; a larger range of available grain sizes might allow stabilization of zones of higher shear stress by formation of a less mobile local coarse bed. The other possible explanation is that the stress variance is controlled in part by the total sediment flux. Local increases in sediment flux are known to lead to periods of more active bar formation and channel migration (Ashmore, 1991); this should lead to a higher rate of formation of confluences, which are the main source of positive local stress fluctuations. We consider it likely that both effects contribute.

The work presented here makes it clear that stochastic effects play a major role in governing total sediment flux through braided river systems; however, estimating the statistical parameters on which the theory depends involves field work that would be impractical in many cases and impossible in the ancient. A more sophisticated theory of spatially averaged transport in braided rivers would allow estimation of the key statistical parameters using only the imposed variables (water and sediment discharges, grain size, basin geometry) and would allow for downstream variation such as indicated in Table 1. To do that, however, we must develop an understanding of how these parameters are controlled by braiding dynamics. Much useful work has been done, for example, on various aspects of channel confluences (Ashmore and Parker, 1983; Ashmore et al., 1992; Best, 1988; Best and Ashworth, 1997; Biron et al., 1993; Brissett et al., 1993; Rhoads and Kenworthy, 1995; Roy and Bergerson, 1990). This work is of particular importance to statistical analyses, such as the one presented here, because confluences contribute disproportionately to overall sediment flux due to the combination of local high stresses and the intrinsic nonlinearity of sediment-transport relations. As valuable as the fine-scale work on confluence structure is, however, little has been added to the pioneering work of Ashmore (1985) on the overall dynamics of confluence formation. Such work would be a substantial step toward a truly predictive theory of overall sediment flux in braided rivers.

**CONCLUSIONS**

1. A formal theory based on decomposition into mean and fluctuating quantities, analogous to the Reynolds decomposition of
turbulent flow, can be used to determine mean behavior of braided-river systems from mean input quantities. There is no reason why the same approach could not be applied to other types of rivers or other transport systems as well.

2. Just as in turbulence theory, the braided-river decomposition leads to introduction of additional statistical quantities, analogous to Reynolds stresses, that must be independently constrained. Just as Reynolds stresses are properties of the flow rather than the fluid, these quantities are properties of the river system and can be expected to vary along a given river and between river systems.

3. Although the terms related to fluctuations in the stochastic theory of rivers do not dominate transport to the extent that they do in turbulence, they nonetheless are important: The mean slope needed to transport the imposed sediment discharge with the imposed water discharge for the study case is about 2.5 times smaller than would be estimated from mean quantities alone.

4. As in the case of turbulence, the decomposition approach presented here is unlikely to lead to a complete, closed physical theory. Detailed prediction of the behavior of braided rivers probably will require a complete and highly resolved model for the coupled flow and sediment-transport fields, analogous to detailed numerical simulations of turbulence; nonetheless, for many purposes (e.g., stratigraphic modeling), semiparametric closures similar to the one developed here may provide results of sufficient accuracy, and would be analogous to the many forms of eddy-viscosity closure still in use for flow problems where detailed turbulence modeling is impractical. The case for a semiparametric approach is strengthened by the observation that although they are important, the fluctuation-related transport terms in the river theory are not overwhelmingly large, as they are in turbulent flows. The success of such an approach depends on accumulating reach-scale statistical data sets for a variety of river types.

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